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Relative-distance Machian theories

MACH'S principle, in essence, requires that the dynamical law of the Universe be expressed ultimately in terms only of the relative distances between the observable entities in the universe. Here I propose a general framework for constructing theories that satisfy this postulate automatically. A simple model shows how the Newtonian world picture can be satisfactorily 'Machianised.' In principle, one could attempt to Machianise the post-1905 world picture along similar lines.

The key Kinematic concept is the relative configuration space (RCS) of the Universe. If, as in Newtonian theory, the Universe is assumed to consist of N particles in a fixed three-dimensional Euclidean space, the points of the RCS (the Newtonian RCS) are all the distinct relative configurations of these N particles. Many other RCSs are conceivable. For example, the points of an RCS could be all distinct three-geometries, and the RCS would then be the 'superspace' familiar in general relativity[1]. The general framework is not, therefore, tied to pre-1905 physics.

A kinematical history of the Universe (or RCS curve) is any continuous curve in the RCS. On any such curve, each point defines an 'instant of time' in the given history; the unfolding of time is but the fact of the universe's moving along some RCS curve (Leibniz's concept of time). Note that time — and therefore the possibility of defining relative velocities — is derived from within the RCS; note also the emphasis that is placed on the motion of the Universe as a whole.

If the dynamical law of the Universe is expressed solely in terms of elements taken from within the RCS, the theory must be Machian by construction. This is the general criterion that a Machian dynamics must satisfy. One possibility of realising such a dynamics is to seek a variational principle that contains only allowed kinematical elements and makes actually realised RCS curves extremal compared with neighbouring kinematically allowed curves. The initial conditions are specified by the direction of the RCS curve at a given point of the RCS. Boundary conditions do not arise.

In the case of the Newtonian RCS, space will play a geometrical but not dynamical role (in contrast to Newtonian mechanics). In a theory in which three-geometries constitute the points of the RCS, a Machian dynamics would automatically entail a variable three-geometry that unfolds in accordance with a Machian law. In this case, it would be more appropriate to speak of a relative-configuration (rather than relative-distance) Machian theory. Such an approach is not identical to the superspace approach to general relativity, in which it is assumed *a priori* that a sequence of three-geometries 'stacks' into a four-dimensional Riemannian space[1]. I see here the extraneous element (not taken from within the RCS) responsible for general relativity's failure to give an entirely satisfactory account of inertia.

It seems difficult to reconcile the universal time defined here with the local time of special relativity. But the flat-space approach to general relativity[2,3] has shown how the *a priori* assumed kinematical elements of a theory can, in fact, be rendered inobservable when a dynamical law of gravitation is imposed on a prescribed kinematical structure. In principle, something similar could happen here — a dynamical law, perhaps in conjunction with a particular class of plausible initial conditions, could lead to a dynamics in which only a local time can in fact be observed. The present approach reverses the priorities that Einstein had in mind when constructing general relativity. Instead of requiring special relativity to hold in the small under all conceivable circumstances in all possible universes (and hoping that this can somehow be reconciled with Mach's principle *via* the equivalence principle), the aim here is to construct theories that are Machian of necessity and then to see if any of these give rise to special relativity in the small in universes like ours.

In the case of a Newtonian RCS with N point particles $i = 1, \ldots, N$ of mass m_i (an intrinsic number associated with each particle; $\Sigma m_i = M$), the obvious allowed kinematical elements are the relative distances $\dot{r}_{ij}(\lambda)$ between all pairs i and j of particles, and their derivatives $r_{ij} = \mathrm{d}r_{ij}/\mathrm{d}\lambda$ with respect to an arbitrary time parameter λ along the RCS curve. The simplest nontrivial gravito-inertial dynamics is then defined by the Lagrange function

$$L = \psi\Gamma \tag{1}$$

where

$$\Gamma = \Big(\sum_{i<j} m_i m_j \dot{r}_{ij}^2\Big)^{1/2}, \quad i = 1, \ldots, N$$

$$\psi = \sum_{i<j} m_i m_j / r_{ij}$$

Equation (1) has a product form to ensure that $L\mathrm{d}\lambda$ is independent of the time parameter (a sum like $(\psi + \Gamma)\mathrm{d}\lambda$ would not be λ-independent).

The equations of motion deduced from (1) in three dimensions are too complicated to be readily comprehensible. Simple equations are obtained in one dimension by introducing Cartesian coordinates, whose origin may execute any suitably continuous motion whatsoever relative to the particles in their one-dimensional universe. The resulting Euler-Lagrange equations will describe the uniquely determined relative motion (which is all that is observable) in terms of the arbitrarily chosen coordinate system. Let $x_i(\lambda)$ be the coordinate of particle i; then L becomes

$$L = \psi\Big[\sum_{i<j} m_i m_j (\dot{x}_i^2 - 2\dot{x}_i\dot{x}_j + \dot{x}_j^2)\Big]^{1/2} \tag{2}$$

and the Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}(\partial L/\partial \dot{x}_i) = \delta L/\partial x_i$$

are

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left\{\frac{\psi m_i}{\Gamma}\Big[\sum_{j \neq i} m_j(\dot{x}_i - \dot{x}_j)\Big]\right\} =$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left\{\frac{\psi m_i}{\Gamma}\Big[\dot{x}_i(M - m_i) - \sum_{j \neq i} m_j\dot{x}_j\Big]\right\} = \Gamma\frac{\partial \psi}{\partial x_i}$$

We can now specialise both λ, taking

$$\mathrm{d}\lambda = \mathrm{d}s = \Big(\sum_{i<j} m_i m_j \mathrm{d}r^2_{ij}\Big)^{1/2}, \quad \text{i.e. } \Gamma = 1,$$

and the coordinate system, taking it such that

$$\Sigma m_i \ddot{x}_i = 0 \ (\text{here } \mathrm{d}\lambda = \mathrm{d}s)$$

In these special frames, which are distinguished by the uniquely determined relative motion, the equations of motion simplify to

$$M\frac{\mathrm{d}}{\mathrm{d}s}(\psi m_i \dot{x}_i) = \frac{\partial \psi}{\partial x_i} \tag{3}$$

If only a few particles are present in the Universe, equations (3) clearly lead to motion that is very different from the Newtonian. On the other hand, in an environment broadly similar to ours (very many stars distributed uniformly over a large region), ψ will be effectively constant and s will be indistinguishable from Newtonian time. Then equation (3) becomes

$$m_i\,\mathrm{d}\dot{x}_i/\mathrm{d}t = (1/M\psi)\partial\psi/\partial\dot{x}_i \tag{4}$$

where $\gamma = 1/M\psi$ is a 'gravitational constant' that is determined by the actual distribution of matter in the Universe. A different model (in preparation) turns out to have more interesting properties, so I shall not attempt to estimate γ here. In the neighbourhood of the Sun, say, the motion predicted by equation (4) is essentially indistinguishable from the Newtonian if γ has the correct value.

This model is, I believe, interesting for several reasons. First, it explains inertia (resistance of a body to rectilinear acceleration relative to the remaining bodies in the Universe) solely in terms of relative distances and relative velocities and demonstrates that a complete dynamics can be expressed in such terms. For the discussion of how Mach's principle should be implemented, this may be important, for Einstein, in particular, doubted whether the Newtonian world picture could be Machianised in this manner[4]. It is particularly satisfying that preferred frames (the inertial frames of Newtonian theory) are distinguished by a motion whose dynamics is expressible solely in terms of relative distances.

Second, although two features of Newtonian gravitation are introduced *a priori* in equation (1) — all gravitational 'charges' are of the same sign and are proportional to the inertial 'masses' — the theory itself determines the actual strength of gravity (because a coupling constant cannot enter the product L) which must be attractive. (In the model in preparation, gravity and inertia are truly subsumed into a single mechanism and the correct order of magnitude of the gravitational constant is predicted.)

Third, whereas Newtonian theory and general relativity provide equations of motion for essentially inobservable quantities in the limit of universes containing only a few particles, no such problem arises in the present approach. If there is just one particle in the Universe, the RCS does not even exist, so there is no theory at all. If there are two particles, the RCS is the positive half axis, but no equation of motion can be formulated because equation (1) becomes trivial; the particles may approach or recede but no meaning can be attached to the rate at which this happens. Nontrivial dynamics first becomes possible when there are three particles, and even then the motion is quite unlike the Newtonian. Many particles in an isotropic background are required to build up the inertial forces as we know them in this universe.

Other forces could be included readily by adding a term linear in the r_{ij}s to L (gyroscopic-type forces) or multiplying L by another factor $\Phi = \Phi(r_{ij})$. Indeed, in an attempt to include electrostatics, take $L' = \Phi\Psi\Gamma$, where $\Phi = \sum_{i<j} e_i e_j / r_{ij}$, with $e_i = 1$ for $i = 1, \ldots N/2$ and $e_i = -1$ for $i = N/2 + 1, \ldots N$. Then the equations of motion corresponding to the conditions under which equation (4) hold are

$$m_i \, d\dot{x}_i/dt = (1/M\Psi) \, \partial\Psi/\partial\dot{x}_i + (1/M\Phi) \, \partial\Phi/\partial\dot{x}_i. \qquad (5)$$

As L' is a product, the relative strength of the new forces is again determined by the theory for each particular universe. In fact, if the particles in this case have equal masses and are distributed more or less uniformly, then $|\Psi/\Phi| \sim N$, because all the $\sim N^2$ terms in Ψ are positive, whereas there are just N more negative than positive terms in Φ. Thus, the electrostatic forces in equation (5) are $\sim N$ times stronger than the gravitational forces and like charges repel and opposites attract (provided $\Phi < 0$; I do not know whether distributions could exist for which $\Phi > 0$). As it happens there are $N \sim 10^{80}$ baryons in the observable Universe[5], whereas we require a strength ratio $\sim 10^{40}$, but the fact that we get the square of the right answer is perhaps an indication that product Lagrangians, which arise naturally (if not uniquely) in such a global approach to motion, could perhaps provide an explanation for the famous cosmic coincidences[5].

If the general framework is in fact the right approach to the problem of inertia, radical changes in theoretical physics are almost certainly necessary, and it is unlikely that the simple model presented here could bear any more resemblance to an ultimately successful theory than Bohr's original model of the hydrogen atom does to modern quantum field theory.

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