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The part played by Mach's Principle in the genesis of relativistic cosmology

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The expression "Mach's Principle" was coined in 1918 by Einstein (Einstein 1918) to denote a principle which he claimed was a "generalization of Mach's requirement that inertia be traced back to an interaction between bodies." Any talk about Mach's Principle is fraught with problems, since it is notoriously difficult to get any two physicists – let alone philosophers – to agree on the precise content of the principle or on the extent of its validity. Since we are concerned here with the birth of modern cosmology, it would be quite inappropriate for me to attempt either an extended critique or a detailed analysis of the extent to which Einstein succeeded in his own Machian aims. Instead, I shall merely allow myself a few comments on why, in my view, discussion about Mach's Principle is so confused and then concentrate on what Einstein took the Machian requirement to be and how the attempt to fulfill this requirement helped him, first, to find his general theory of relativity, and then, in the framework of that theory, to construct in 1917 the first scientifically based model of the universe, creating simultaneously in a brief paper of just eleven pages the strikingly bold framework of relativistic cosmology.

Let me first make a general remark. Cosmology is a unique subject and, perhaps more than any other, forces us to consider the question of foundations. The viewpoint that Mach expressed so cogently and influentially was that there are in fact no external foundations. There can be no foundation on which the world rests. The observed world must itself supply the terms of its own description. It must be conceived to reside and unfold *self-referentially* in nothing. The Machian requirement that Einstein so intensely felt he had to meet sprang from such a conviction. Through it he wanted, in his own expression (Einstein 1922: 62), to close "the series of causes of mechanical phenomena." He wanted to construct a causally self-contained world with observable causes of all observable effects.

In this connection, before turning to details, I should like to draw attention to a much earlier occasion in the history of physics when distinctly Machian ideas played a role that was especially relevant for the eventual emergence of relativistic cosmology. I am referring to the discovery by Kepler in the period 1600 to 1605 of his first two laws of planetary motion. These were, of course, crucial for the discovery by Newton of his law of universal gravitation, without which there could clearly never have been either the general theory of relativity or relativistic cosmology. Careful reading of Kepler's *Astronomia Nova* (1609), in which he recounted how he made his great discoveries, reveals many remarkable parallels with Machian attitudes to the basic problems of motion. The similarity is not at all fortuitous and is closely related to the point made in the last paragraph, namely, that the world and its parts do not rest on any "external" foundations. Mach rejected the notion of an *invisible* background space as the agent responsible for the phenomenon of inertial motion and was therefore forced to seek some other real (and *visible*) agent to fulfill this crucial role. In his time, the distant masses of the universe were the only natural candidates, and this led to his conjecture that they, and not absolute space, were what guided local bodies in their inertial motions. But, centuries earlier, Kepler was forced to embark on a remarkably similar quest for the ultimate determinants of observed motions by Brahe's assertion, deduced from the apparent motions of the comets, that the crystal spheres hitherto supposed to carry the planets simply could not exist. Kepler completely accepted Brahe's argument and it profoundly influenced his attitude to the problem of the planetary motions. For it appeared that the planets must somehow find their way through the completely featureless ether of interplanetary space along quite definite paths but without anything to guide them or move them. He was forced to develop a conceptual scheme in which the planets's motions were directly determined (both generated and guided) by the bodies known to exist in the universe, above all the Sun and the stars. This makes the parallel with the Machian program evident. In its time, Kepler's basic approach was every bit as radical and revolutionary as Mach's much later proposal. For in the framework of the ancient astronomical techniques which Kepler inherited from Copernicus the vital role of the Sun in governing the planetary motions was completely hidden. (For very understandable reasons, which have to do with ancient astronomical traditions and the specific eccentricities of the various planetary orbits, Copernicus actually centered the entire planetary system on the void second focus of the Earth's orbit. The Sun is not even shown in any of his diagrams explaining the motions of the planets!) Moreover, Kepler's "Mach's Principle" was carried through to successful implementation in the form of his first two laws of planetary motion, whereas Mach only made his proposal but took it no further himself. It would be very easy to say a great deal more about this intriguing parallel, including also some very striking similarities between Kepler's long Odyssey in search of the laws of planetary motion and Einstein's equally long one in search of his general theory of relativity, but that would leave no time at all for my main topic and I have written about the subject elsewhere (Barbour 1989: chapter 6). Here, I shall say only that Kepler's achievements, which were not properly appreciated for over half a century, were recognized very early in the University of Bologna, at which there seems to have been "a more or less continuous Keplerian tradition" (Russell 1964) spanning most of the period between the discovery by Kepler of his laws and their dynamical interpretation by Newton three-quarters of a century later.

Coming now to the main topic, let me first mention and, I hope, dispose of the prime source of confusion on the subject of Mach's Principle. It is Einstein's curious assertion, first made in 1912 (Einstein 1912) and then repeated or implied on numerous occasions in his subsequent writings (for example, Einstein and Grossmann 1913; Einstein 1917) that Mach wished to establish the *relativity of inertia*. By *inertia* in this context Einstein quite definitely meant the inertial mass that appears in Newton's second law, i.e., the coefficient that multiplies the acceleration. It is not too difficult to see how Einstein came to believe he could explain inertial resistance to acceleration (his argument will be sketched near the end of this paper), but I am still at a loss to understand why Einstein thought that Mach sought to establish relativity of inertia in such a sense. There is nothing in Mach's writings to suggest it; quite the opposite. I almost think that on this point Einstein was the victim of a semantic confusion. For Mach, who, after all, created a beautiful operational definition of inertial mass (Mach 1960: 264ff), saw no problem at all in that use of the word inertia. What always concerned him were concepts at a much more primitive level, those of position and velocity. His concern was not with *inertial resistance* but with the *law of inertia* (Mach 1960: 271ff). He was concerned solely with what is often called *kinematic relativity*. Bodies can be observed only relative to other bodies. In a universe in which all bodies are in a state of relative motion, how can objective meaning be given to the idea of any definite motion, let alone a uniform one in a straight line? It was this problem that made Newton introduce the notions of absolute space and time.

If we examine Einstein's work closely, we see that in reality his notion of relativity of inertia did not significantly influence the basic structure of the general theory of relativity, though, as we shall see, it did decisively influence the construction of his cosmological model (and, perhaps even more strongly, de Sitter's rival model). The central problem in the creation of the general theory of relativity did not revolve around relativity of inertial mass but around *relativity of frames of reference*. There is no doubt that in addressing the question of frames of reference and the fact that some appeared to be distinguished compared with others Einstein was addressing the same problem as Mach. However, even here there was a very important difference of approach that was brought about by two factors. The first was the evolution of ideas during the more than thirty years which elapsed between Mach's original questioning of the law of inertia and Einstein's attempt to do something about it. The second was the dramatic effect of what Einstein was later to call "the happiest thought of my life" – the discovery of the equivalence principle.[1] This had a most profound influence on the way in which Einstein attacked the problem of the frames of reference.

Einstein defined an inertial frame of reference, which was absolutely central to his special theory of relativity, either as a frame in which Newton's laws were found to hold or, more vaguely and more generally, as one in which the laws of nature were found to take their simplest form. For quite some time their existence had been felt to be paradoxical in view of the manifest relativity of motion that Mach, above all, stressed so strongly. Although Mach himself produced no concrete theory of the origin of these mysterious frames of reference, two very characteristic remarks of his give a pretty clear hint of the direction in which he was thinking. One was made in connection with his criticism of Newton's bucket experiment (Mach 1960: 284): "The universe is not *twice* given with an earth at rest and an earth in motion; but only *once*, with its *relative* motions, alone determinable." The second remark (p. 286) was: "When we say that a body preserves unchanged its direction and velocity *in space*, our assertion is nothing more or less than an abbreviated reference to *the entire universe*." I would like to draw special attention to the great prominence given in these remarks to the universe as a whole, which Mach evidently regarded as a single dynamical entity. Mach reformulated the aim of dynamics: it was not to provide laws of motion of individual bodies in space and time but rather to study the evolution in time of the relative separations of the bodies of the universe. In modern terms, the fundamental dynamical variables are not position vectors of bodies in space but all the relative distances between the various bodies in the universe.

The logic of Mach's approach was to dispense altogether with coordinate systems and frames of reference and concentrate instead directly on the *universe as a whole*, describing it by a relational law containing only relative distances and relative velocities. Distinguished frames of reference would then arise only if we fix our attention on local bodies and attempt to describe them in coordinate systems chosen to make their motion appear particularly simple, as in Newton's first law. Since the motions are in reality taking place with respect to the universe at large, our distinguished coordinate systems will actually be tied to and determined by the same universe. This, in essence, was the Machian explanation for the

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[1] For the charming account of how it happened, see Pais (1982: 177ff).

observed coincidence of the family of inertial frames with frames nonrotating with respect to the distant stars. In such an approach one must clearly distinguish between a basic, let us call it primordial, relational law of the universe as a whole and effective local laws that are recovered from the primordial law by referring local motions to special frames. Clearly, the first step, which Mach never took unfortunately, was to find the primordial law of the universe as a whole.[2]

I have sketched this Machian approach to highlight the fact that Einstein adopted a totally different one. This I believe is a second major reason why discussion of Mach's Principle is so tangled. Although there are several quite clear hints in Einstein's papers that local physics is crucially influenced by the matter in the universe at large (especially in Einstein 1912, 1916) they remained only implicit (in the background inspiration), and the *explicit* consideration of the universe as a single dynamical entity occurs remarkably late in Einstein's papers and only *after* his theory was essentially complete. As we shall see, this had an important consequence: whereas in the period up to 1916 Einstein believed that he was implementing Mach's Principle automatically and dynamically, by the very structure of his equations, in the later period, when the dynamical structure of his theory was already complete, he was forced to attempt to implement his Machian idea by means of boundary conditions.

To highlight the lateness at which Einstein turned his attention seriously, that is, as a matter of first priority, to the relationship between the local dynamics and the universe at large, let me remind you of Aristotle's spherical cosmos, created well over 2,000 years before Einstein's. The similarities between the two are really rather striking: both are spatially spherical and self-contained and both extend in time infinitely far into the past and the future. Both exist in nothing. Aristotle is careful to point out that outside his cosmos there exists nothing at all – not even space or time. What is especially interesting is that both cosmological models – Aristotle's and Einstein's – were created in response to what may be called Machian considerations. There was, in fact, a most interesting and illuminating pre-run of the absolute/relative debate in antiquity. It was stimulated very largely by the atomists. Aristotle was unhappy about their idea of atoms in a void and often criticized the void in distinctly Machian terms (especially in his *On the Heavens* and *Physics*; for a fuller discussion of these questions and detailed references, see Barbour 1989). His cosmology, with its well-defined outer shell and equally well-defined center was conceived explicitly to provide a framework for his laws of motion, just as Newton invoked absolute space. Thus, in Aristotle's case the form of his cosmos and the specific laws of motion crystallized together. The relation between the two entered explicitly from the

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[2] Bertotti and I have shown how a Machian program of the kind just outlined can be carried out in detail (Barbour 1974; Barbour and Bertotti 1977, 1982).

beginning. In Einstein's case it was quite different – the laws of motion came first, the explicit cosmological considerations very late.

It is not hard to find reasons for this difference. Jacques Merleau-Ponty (Merleau-Ponty 1982) has noted that throughout the nineteenth century cosmology was almost totally ignored by the physics and astronomy community. A rather positivistic approach to the natural sciences was adopted – it was all to do with measurement of the properties of matter as observed around us and description of the results obtained by mathematics by a process of induction. I believe a most important factor in this connection was the practical success of Newton's concepts of absolute space and time, particularly after they had been made epistemologically more respectable by the concepts of inertial frames of reference. In fact, absolute space and time provided a kind of surrogate cosmology and this is what made it possible for serious concern with cosmological questions to be deferred for such a remarkably long time.

A final point to be made about the difference of approach between Einstein and Mach is that one can find remarkably little explicit concern anywhere in Einstein's writings for the issues relating to the *relativity of motion* that loom so large in Mach's writings. The problem I have referred to as kinematic relativity is seldom, if ever, directly addressed or even mentioned. Einstein simply does not consider the question of the practical determination of position of a given body by means of the other bodies in the universe. In contrast to his contemporary Weyl, for example, he hardly uses the expression "relativity of motion" (he almost always refers to "relativity of inertia"). In fact, Einstein circumvented the basic problems of position determination by his use, from the very beginning, of the concept of *frames of reference*, and he attacked the problem of their distinguished nature, not along the lines just outlined, i.e., from a primordial relational law, but by questioning whether they were distinguished at all. Moreover, for a very long time his outlook remained essentially local – he attempted to abolish the distinction at a local level. There are no doubt several good historical reasons that can explain Einstein's preference for a local rather than a global approach. First among these must obviously have been the rejection of instantaneous action at a distance and the associated rise of field theory, with which Einstein himself was so closely associated. In addition, there was the general absence of direct concern with cosmology just mentioned. However, equally if not more important was the totally new principle that Einstein found – the equivalence principle. I have said enough about what Mach would have done and Einstein might have done. It is time to consider what Einstein actually did.

His overall strategy, from which he never really wavered until after the theoretical structure of general relativity was complete, is clearly revealed in his first comments on the subject of gravitation in 1907. At the end of a review of the special theory of relativity (Einstein 1907), he commented that hitherto one had required the laws of nature to be independent of the state of motion only for *unaccelerated* frames of reference. He then asked: "Could one suppose that the principle of relativity is also satisfied for systems moving relatively to each other with acceleration?" He added that "This question must occur to anyone who has followed the applications of the principle of relativity up to the present time." It was, of course, the equivalence principle, only given that name a few years later, that enabled Einstein to pose such a seemingly absurd question. From the fact that "in a gravitational field all bodies are accelerated equally" Einstein argued that "at the present state of our knowledge" there are no grounds for believing that there are any respects in which a frame of reference accelerated uniformly in a region free of a gravitational field differs from one at rest in a homogeneous gravitational field.

Discussion of Einstein's work has tended to concentrate on the way in which he used the equivalence principle to draw some first conclusions about the laws of physical processes in homogeneous gravitational fields. But the fact that he was from the start simultaneously following a further aim becomes clear from his second paper on the subject, published in 1911. There he said (Einstein 1911): "In such an approach one cannot speak of the absolute acceleration of the coordinate system any more than in the special theory of relativity one can speak of the *absolute velocity* of the system" (original italics).

The drift of Einstein's thought is now clear. Whereas the logic of Mach's comments called for explicit derivation of the distinguished local frames of reference from a relational law of the cosmos as a whole, Einstein is working towards elimination of the problem of the distinguished frames by asserting that they are not really distinguished at all. The first step, more or less successfully accomplished in the earliest papers, was to dispose at least of the frames in a state of uniform acceleration.

But the success was in fact paid for at a price and this price casts a lot of light on the problems Einstein faced in his approach to the Machian problem. For the original relativity principle purports to be a universal principle, that is, it asserts that all physical processes unfold in exactly the same way in all the allowed equivalent frames of reference. However, in the case of Einstein's extension to uniformly accelerated frames of reference or homogeneous gravitational fields the laws of the gravitational field itself are simply not covered. The principle merely gives one information about the way in which nongravitational processes unfold in a given and, in fact, homogeneous gravitational field. Whereas in the case of the original principle of relativity external factors were of no concern, the success of Einstein's extension depends crucially on things outside the system.

There is an essential incompleteness in all of Einstein's attempts to explain the effects of so-called inertial forces in terms of a gravitational field. Indeed, I do not think Einstein ever addressed this question thoroughly and directly. He never attempted to spell out in explicit detail precisely how the universe at large produced the particular gravitational field that would permit him to say that all apparently inertial effects are really gravitational and that therefore distinguished frames do not occur at all. This was a problem that, probably wisely, Einstein kept on deferring.

The equivalence principle and the scope he saw in it for solving the Machian problem were above all important for Einstein because they gave him the confidence to think the unthinkable.

One of the clearest examples of this can be seen in what Stachel (1980) has called the "missing link," namely, the precise insight that guided Einstein to the notion of a four-dimensional Riemannian manifold with a genuinely non-Euclidean geometry. A passage in a letter to Sommerfeld written in September 1909 shows which consideration above all it was that kept moving him forward into new territory. He wrote (Stachel 1980): "The treatment of the uniformly rotating rigid body seems to me to be of great importance on account of an extension of the relativity principle to uniformly rotating systems along analogous lines of thought to those that I tried to carry out for uniformly accelerated translation in the last section of my paper published in the *Zeitschrift für Radioaktivität*." Einstein was probably prompted to this remark by a "paradox" that had just been discovered by Ehrenfest when he attempted to consider the geometry of a rotating disk in the framework of special relativity. For the length of a rod placed radially on a rotating disk should not undergo any Lorentz–Fitzgerald contraction relative to a nonrotating frame of reference because it would always be moving transversely. However, rods placed around the rim should be subject to a contraction, and therefore such rods would reveal a circumference of the circle that was more than π times the diameter. Stachel has marshalled evidence which shows that Einstein took this result, with its implications of the need to consider non-Euclidean geometry, very seriously. He had to, since it was his serious intention to show that all frames of reference should be equally valid for the description of nature.

Einstein's conclusion from the example of a rotating disk of the need to consider non-Euclidean geometry is rather ironic, since in the light of mature general relativity he had actually drawn an invalid conclusion. This came about because Einstein at that time used the concepts of frames of reference and transformations between them in a manner that does not correspond to the correct transformation laws of tensor calculus in general relativity.[3] Specifically, he was attempting to measure the geometry in a

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[3] Norton (1984) makes some interesting comments on Einstein's use of frames of reference in his detailed study of Einstein's discovery of the equations of general relativity. His paper contains references to all of Einstein's important papers on general relativity in the period 1912 to 1915 together with unpublished material and letters.

spacelike hypersurface with measuring rods that do not move orthogonally to it in four-dimensional space-time. As a result, he generated a spurious non-Euclidean geometry of the instantaneous spacelike hypersurfaces, which must, of course, remain flat in both the original frame of reference and the rotating frame. However, this error, if we can call it such, was extremely helpful.

The definitive breakthrough on Einstein's part in the summer of 1912 to a four-dimensional space-time manifold with non-Euclidean geometry undoubtedly owed much to non-Machian factors but here again the possibility of extending the relativity principle to all frames of reference seems to have been the most important reason for Einstein's belief that he had at last found the correct framework. The key idea was his ansatz for the law of motion of a test particle in the form of the geodesic principle

$$\delta \int ds = 0, \; ds = (g_{\mu\nu} dx^{\mu} dx^{\nu})^{\frac{1}{2}},$$

where $g_{\mu\nu}$ is the metric tensor of the manifold, and his realization that such a law would take the identical form in absolutely any system of coordinates (Einstein and Grossmann 1913). In several places Einstein expressed the opinion that by itself this result already eliminated the problem of the distinguished frames of reference (see, for example, Einstein 1914). Einstein's concept of the Machian requirement had now become much more precise but it also, I believe, underwent a certain modification of content which prepared the way for Einstein's acceptance in 1918 (Einstein 1918) of Kretschmann's argument (Kretschmann 1917) that general covariance had no physical content but was merely a formal requirement of mutually consistent description of a unique object from different points of view.

Despite his assertion that his general framework by itself solved the Machian problem, Einstein was very well aware that he had solved only half of his problem. It was also necessary to find the equations of the gravitational field itself. We have here the remarkable story of Einstein's initial instinctive belief that they too must be generally covariant, the amazingly near miss on the part of Grossmann and himself when they attempted in their first joint paper in 1913 to find satisfactory field equations of such form (Einstein and Grossmann 1913), and then the invention by Einstein of his notorious argument by which he attempted to prove that the field equations of the general theory of relativity, the name he already gave to his incipient theory, could not themselves be generally covariant. This is a most tangled story into which it would be impossible to delve deeply here.[4] All that I will say is that despite all his difficulties and mistakes Einstein could never get the ideal of complete general covariance out of his head. He was quite capable of giving a proof that his current equations with only restricted covariance must be correct but then almost

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[4] For a detailed discussion, see Norton (1984) and also Earman and Glymour (1978).

with the same breath saying the theory should be completely covariant. And the reason for the attachment to general covariance was clear – it alone would guarantee realization of his Machian ideal.

To cut a long story short, Einstein did finally return to general covariance and by the end of November 1915 had at last completed his great work, the general theory of relativity. In the famous summary of his theory published in 1916 in the *Annalen der Physik* (Einstein 1916) the very greatest emphasis is placed on the need for complete general relativity in order to resolve the problem of distinguished frames of reference. At the conclusion of his discussion of his famous example of two liquid spheres in a state of relative rotation, the one flattened but the other not, Einstein says:

> Of all imaginable spaces ... in any kind of motion relatively to one another, there is none which we may look upon as privileged *a priori* without reviving the above-mentioned epistemological objection. *The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion.*

This statement marks the apotheosis of the relativity principle in its role as implementation of the Machian requirement directly and dynamically through the basic structure of the theory. It is the *point omega* towards which Einstein had been working methodically for nearly nine years – yet two years later he was to concede that he had not distinguished sufficiently clearly between the relativity principle and the Machian requirement and he was led to introduce a quite separate Mach's Principle (Einstein 1918). Before we consider this surprising turn and its consequences, a brief review is appropriate. Precisely how important was the Machian factor to Einstein as compared with the numerous other strands that he wove together in his theory? One must here distinguish between tools used to do a job and the inspiration to undertake it. Dealing with the tools first, I was very struck when going through all the Einstein papers just how many solid and well-established results from modern physics and mathematics Einstein did use – and, moreover, how effectively he used them. What is almost breathtaking and lends his theory such grandeur is the way in which he consistently applied the lesson he had learnt from special relativity – namely, to achieve the result you want, do not be afraid to tamper with space and time. In fact, it seems to me that Einstein exhibited an almost ruthless willingness to do just whatever he pleased to space and time provided only he could then show that the laws of nature would take exactly the same form at every point of space-time and in any frame of reference in which he might care to examine them. In terms of the concrete steps taken and the actual tools used in the process, special relativity and a whole slew of results that came with it – above all ones relating to Maxwell's theory and Minkowski's space-time formulation – were vastly more effective in establishing the final shape of the creation than was the Machian ideal. But if we ask how it was possible that Einstein ever came to create such an incredible theory, so utterly unlike anything even his most brilliant contemporaries were prepared to consider, the answer is clear. It was the Machian inspiration, never really precisely grasped, that provided the touch of magic and constantly drew him on. In the words of Keats, it was a case of "Heard melodies are sweet, but those unheard are sweeter."

Now we come to the cosmology. The manner in which Einstein wrote the introduction to the review of the *Annalen der Physik* indicates that at that time he still believed the Machian requirement was automatically satisfied by the very structure of the theory, i.e., by its general covariance. The first seeds of doubt arose from reflection on the solutions to his equations by means of which the planetary motions were described. For in these solutions the $g_{\mu\nu}$ were assumed to tend at infinity to the Galilean values

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

the local concentration of matter in the sun merely giving rise to rather insignificant distortions in its vicinity. It seems to have dawned on Einstein rather slowly that the Machian ideal which had sustained him through such travail appeared to be almost as remote as ever.

This brings us to the final part of the story and Einstein's last attempt to grasp his goal. This attempt too was just as ironic as the first failed attempt. Yet again there was no implementation of Mach's Principle but once again there was a momentous consequence. This time it was the science of relativistic cosmology. It was born because, for the very first time in his work, Einstein addressed himself directly to the undertaking that had from the beginning been implicit in the Machian enterprise, namely, the derivation of local physics from the dynamics of the universe as a whole.

At this stage of the story Einstein's notion of the relativity of inertial mass became really decisive and truly dictated the steps he took. He expressed his article of faith in these words in his 1917 paper on cosmology (Einstein 1917): "In a consistent theory of relativity there can be no inertia *relatively to 'space*,' but only an inertia of masses *relatively to one another*." From this, Einstein drew an important conclusion: "If, therefore, I have a mass at a sufficient distance from all other masses in the universe, its inertia must fall to zero." This single remark was to give rise to not only Einstein's cosmological model but also de Sitter's, as we shall shortly see.

Einstein's first idea, which he was developing in September 1916 at the time of a visit to Leiden in Holland, where he had several very important discussions with de Sitter, was to find a solution for his equations such that sufficiently far from all matter, i.e., at infinity, the criterion just formulated should hold.

He noted that in the general theory of relativity the equation of motion of a particle indicated that for a particle of rest mass m and 4-velocity dx^α/ds the quantity

$$m\sqrt{-g}\, g_{\mu\nu}(dx^\nu/ds)$$

can be identified as the negative momentum of the particle. Einstein then supposed the case of a spatially isotropic metric and wrote the line element in the form

$$ds^2 = -A(dx_1^2 + dx_2^2 + dx_3^2) + B dx_4^2.$$

From this he concluded that the momentum of the particle would, for small velocities, be proportional to

$$mA/\sqrt{B},$$

so that $mA/\sqrt{B}$ was a measure of the inertia of the particle. He now wanted to achieve that this inertia would tend to zero at spatial infinity. He imposed the coordinate condition $\sqrt{-g} = 1$ and concluded then that A must diminish to zero while B should tend to infinity. He also considered more general situations without assumption of spatial isotropy of the metric, and, as de Sitter reports, contemplated general boundary conditions at infinity in space and time of the symbolic form

$$\begin{pmatrix} 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty^2 \end{pmatrix}.$$

For Einstein, the virtue of such values of the metric was that they were invariant for all transformations $x_\mu \rightarrow x'_\mu$ for which at infinity x_4 is a pure function of x'_4. Such boundary conditions would thus come very close to preserving the general covariance of the field equations. According to de Sitter,[5] Einstein took such boundary conditions very seriously at the time of his visit to Holland. He called them *natural values*.

Back in Germany, Einstein attempted with the help of the mathematician Grommer to see if any realistic matter distribution could lead to the

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[5] De Sitter published several important papers in 1916/1917 (de Sitter 1916, 1917). Besides their intrinsic value in giving the de Sitter solution (1917b, 1917c) they were instrumental in acquainting English scientists with Einstein's theory and also cast a very interesting light on Einstein's approach to Mach's Principle at the time.

metric he required. He did this by the simple device of assuming the metric he required and then calculating from it the energy–momentum tensor that must correspond to it in accordance with the field equations. However, the matter distribution which Einstein and Grommer obtained seemed to be in crass contradiction with the astronomical facts as then known. The status of the nebulae later clearly recognized to be island universes like our own Galaxy and endowed with remarkable velocities of recession was still very obscure (this topic is amply covered by other contributors to this volume), and the galaxies played no part at all in Einstein's thoughts at that time. In fact, his "universe" was effectively not much larger than our Galaxy and its dominant feature, to which Einstein attached much importance in his 1917 paper, appeared to be the remarkably small (compared with the velocity of light) velocities of the stars relative to each other. Einstein concluded from this that, in a suitable frame of reference, the energy–momentum tensor of the matter in the universe must have all components very small compared with the energy density (the 44 component). This did not at all agree with the theoretical requirements that followed from his and Grommer's calculations, and Einstein, seriously misled by the then readily available astronomical knowledge, abandoned such an approach altogether. However, the boundary condition idea did have an important influence on de Sitter, as we shall see.

Einstein then hit upon the idea of doing without boundary conditions altogether. "For," he said, "if it were possible to regard the universe as a continuum which is *finite* (*closed*) *with respect to its spatial dimensions*, we should have no need at all of any such boundary conditions." Thus, we have the explicit formulation of the idea that the universe is a completely self-contained entity and everything that happens within it exists merely in relation to other happenings. All exists truly within nothing.

I shall not attempt to describe in detail Einstein's cosmological model, which he published in February 1917 but mention merely one or two key aspects. First, to construct it at all, Einstein was forced to introduce his famous cosmological term by adding to the field equations

$$G_{\mu\nu} = \varkappa T_{\mu\nu}$$

the term $-\lambda g_{\mu\nu}$ on the left-hand side with an undetermined coefficient λ. The aim of this term, which introduced an effective repulsion of matter, was to ensure that the equations could not have a reasonable cosmological solution without the presence of matter. The necessary presence of matter would then ensure, in Einstein's opinion, that inertia was not merely influenced by matter but completely determined by it. Simultaneously, the presence of the cosmological term would counteract the gravitational attraction of the matter and permit a stable solution. The resulting solution, which only later was recognized to be unstable, represented a finite spherical world in its spatial dimensions but existed from the infinite past to the infinite future in time. It was therefore called the cylindrical world.

Although the model was explicitly constructed with the primary aim of realizing the Machian requirement, one cannot help marvelling when reading the rather brief paper how much of the basic structure of modern relativistic cosmology is contained in it either explicitly or implicitly. I suspect de Sitter deserves a fair measure of credit for this. One gets the impression that in his discussions with Einstein he shifted him towards a scheme that was simultaneously realistic (for its time) and extremely fruitful. Although Einstein apologized to the reader for taking a "rather rough and winding road," the paper is wonderfully lucid and elementary. One wonders if a complete new science was ever created with such effortless ease and, moreover, as a mere by-product. Of course, the work of Riemann and other geometers, coupled with the new dynamical theory, was what made it possible, and the consideration of cosmological models that exploit closed spherical spaces was nothing new. In fact, the youthful Schwarzschild wrote a beautiful little paper on the subject in 1900 (Schwarzschild 1900), which, since it is referred to by de Sitter in one of his papers, may well have had an important influence on de Sitter and, through him (or directly), on Einstein.

What the 1917 paper does show is how naturally the complete structure of relativistic cosmology appeared as soon as there was a genuine stimulus to consider a topic that, prior to the scientific revolution, had been as central to scientific thought as it is possible to imagine but had then been banished to the periphery for about two centuries while scientists busied themselves with seemingly more mundane matters. There is a rather pleasing pattern to the whole period from Aristotle to Einstein – from the one spherical cosmology to the other. Ancient science began with cosmology, which provided a conceptual framework in which solid results of genuine science were gradually accumulated. Eventually, these results blew the old cosmology to pieces, and for a long time scientists worked away in their laboratories without turning round to consider the wider world. No one concentrated more intently on the local laws of nature than Einstein himself. Finally, Mach's insistence on the ultimate question – whence comes the basis of all these local laws? – forced Einstein to look wider and, almost miraculously, all the pieces came together in a general cosmological framework that was every bit as pleasing as the old one but, in contrast to it, had been built from the ground up, not the heavens down. What is especially interesting in the present connection is that all three of the cosmological frameworks that have dominated scientific enquiry in the entire period of its existence – Aristotle's cosmos, Newton's absolute space and time, and relativistic cosmology – crystallized out of deep consideration of the nature of motion (for a detailed discussion of the arguments that guided Aristotle and Newton see Barbour 1989: chapters 3 and 11).

I must now come to the final part of the story. Whereas Einstein's model proved, because of its instability, to have only transitory interest as an actual model of the universe – which in no way diminished its value as a paradigm of model construction – the model which de Sitter was led to construct through his explicit distrust of Einstein's dream of the "relativity of inertia," has proved to be one of the most important in the history of modern cosmology, in particular, for example, in steady-state theory, and, much more recently and still very topically, in the inflation hypothesis.

De Sitter's point of departure was the boundary conditions (display above, p. 58) that Einstein was considering in September 1916. De Sitter had several reservations about Einstein's whole approach. He thought the Machian "relativity of inertia" was a will-o'-the-wisp with which one could dispense without any detriment to the general theory. He disliked the extraordinary masses that at the time Einstein was forced to invoke. He called them "supernatural masses" and "quite as objectionable as absolute space." He was also disturbed by the unsymmetrical treatment of space and time forced upon Einstein by his Machian requirement, according to which the inertia of a particle should become zero at spatial infinity, at which particular boundary conditions needed to be imposed, whereas there was no analogous requirement at temporal infinity. This seemed to him to go right against the spirit of relativity theory. He felt that the only natural and appropriate boundary conditions which one could impose were that the $g_{\mu\nu}$ should vanish at both spatial and temporal infinity. De Sitter was therefore thinking in terms of a cosmological model with a space-time symmetry of a higher order than the one towards which Einstein was working. Indeed, the fact that he found a solution at all, especially one having the very high degree of symmetry which makes it such a remarkable (if not to say mysterious) geometrical entity and world model (cf. Schrödinger 1956) is a direct result of de Sitter's reaction to the incomplete, Machian-dictated symmetry of Einstein's solution. De Sitter was fully conscious of this, and his papers not only gave the solution but also initiated the study of its properties.

Einstein and de Sitter remained in touch and almost as soon as Einstein had constructed his cosmological model de Sitter produced his rival model. It was presented in a paper (de Sitter 1917b, expanded account in de Sitter 1917c) only seven weeks later than Einstein's.

The key idea of the paper, which de Sitter credited to Ehrenfest, was that of making "the four-dimensional world spherical in order to avoid the necessity of assigning boundary conditions." De Sitter's world was actually hyperbolical but by employing an imaginary time coordinate he made it formally spherical to bring out a very close parallel between the cylindrical model of Einstein with its three-dimensional spherical space. Einstein had embedded his three-dimensional space as the surface of a sphere in a four-dimensional Euclidean space. De Sitter noted that if as coordinates to express Einstein's solution one employed the stereographic projections in this higher dimensional space then the metric at infinity tended to the values

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In order to eliminate the asymmetrical and hence objectionable unity in the 44 position, de Sitter obtained his model by embedding a four-dimensional hypersphere in a five-dimensional space. He then obtained a solution in which all the components of the metric in the stereographic projections had the values zero at infinity. He further emphasized the analogy between the two models by giving the explicit parallel expressions for the metric. For Einstein's model the metric was

$$g_{ij} = -\delta_{ij} - \frac{x_i x_j}{R^2 - \Sigma x_i^2},\ g_{44} = 1,\ g_{0i} = 0,\ \Sigma x_i^2 = x_1^2 + x_2^2 + x_3^2,$$

while for his own it was

$$g_{\mu\nu} = -\delta_{\mu\nu} - \frac{x_\mu x_\nu}{R^2 - \Sigma x_\mu^2},\ \Sigma x_\mu^2 = x_1^2 + x_2^2 +{}_3^2 + x_4^2.$$

It should be noted that the extremely close parallel is a little spurious, having been achieved by the use of one imaginary component. However, it does emphasize the way in which the de Sitter solution arose as an explicit reaction against Einstein's Machian requirement.

For models with the very high degree of symmetry assumed by Einstein and de Sitter it is, of course, comparatively easy to solve the field equations. In fact, what the field equations did for both Einstein and de Sitter was to indicate what (averaged) matter was needed to permit the space-time geometry required by the respective models. The answer that de Sitter found in his case was truly the final death knell for Einstein's attempts to implement Mach's Principle. For he found "the remarkable result, that now no 'world-matter' is required." His four-dimensional analogue of the Einstein cosmos did not contain any matter at all. It was completely empty! But for Einstein the whole point of the introduction of the cosmological term had been to enforce the presence of such matter.

Before concluding I should mention the two remarkable papers published by the Russian Friedmann[6] in the *Zeitschrift für Physik* in 1922 and 1924 (Friedmann 1922, 1924). Together with the work we have just discussed, these two papers must surely rank as among the most important ever published in the field of cosmology. Friedmann's work, like that of Einstein and de Sitter, belongs to the more purely theoretical stage of relativistic cosmology that predates the central concern with galactic redshifts. (The redshifts started to become a major topic of discussion between the publication of Friedmann's two papers but are not discussed by him.) Although Friedmann refers to the Machian problem only to the extent of a passing remark that "the problem of centrifugal force" might cast light on the problem of choosing between the numerous different world models that his analysis had revealed as being possible, his work is to be seen as a very natural extension of the model building of Einstein and de Sitter. The overall pattern of both these papers clearly owes a very great deal to their work, and Friedmann says explicitly that his aim is to generalize their work and find solutions to Einstein's equations, (including Einstein's and de Sitter's as special cases), in which the curvature with respect to three coordinates, which serve as space coordinates, is constant with respect to these coordinates but is a function of the fourth, which serves as the time. I think it is therefore fair to say that the last positive service performed by Mach's Principle was that it helped to bring forth, yet again as a by-product, cosmological models that were explicitly evolutionary.[7] This is perhaps the most distinctive feature of modern cosmology. Incidentally, Friedmann's papers really emphasize to an extraordinary degree the point that I made at the start about the universe being conceived to unfold self-referentially in nothing. Rather over half way through Friedmann's first paper, which is written throughout in a very matter-of-fact way but with a lucidity worthy of Einstein, there comes an innocent sounding little definition which must, I think, bring any reader up with a jolt. The only concession to effect that Friedmann makes is to put the concept he is defining in the expanded type often used for a definiendum in German. He considers the time of increase of the radius R of the world from the value 0 to some given radius R_0 and calls it *die Zeit seit der Erschaffung der Welt* – the time since the creation of the world. The cosmos that Friedmann described is completely self-contained yet evolves entirely lawfully. It surely resides in nothing. It even springs out of nothing.

Let me go back to de Sitter's paper. It carried a title with possibly the

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[6] The editors of the *Zeitschrift für Physik* generated some confusion about the spelling of the surname: Friedman in 1922, Friedmann in 1924. Today the transliteration from Russian into English would be Fridman. All three spellings can be found, but Friedmann is the most common.

[7] It seems to me quite likely that Friedmann was stimulated to his study by the almost casual sentence with which Einstein ended his 1917 paper: "It should however be emphasized that a positive curvature of space still results from the presence of matter in it even when the extra term [the cosmological term] is not added; we need that term only to permit a quasistatic distribution of matter, as corresponds to the fact of the small velocities of the stars."

very slightest hint that Einstein was too prone to speculation, namely: "On the relativity of inertia. Remarks concerning Einstein's latest hypothesis." It would in fact be an interesting exercise to count up precisely how many different fruitful hypotheses about gravitation and inertia Einstein did advance between 1907 and 1917. One thing at least is certain: de Sitter's paper on March 31 1917 brought the stream of hypotheses to a definite end. Einstein did not after that add anything of enduring significance to the fundamentals of his gravitational theory, and the long-running saga of Mach's Principle finally petered out into almost nothing, since Einstein was forced to admit that he had not achieved his aim of showing that a cosmological model without matter was impossible. Only gradually could he bring himself to abandon the dream. In fact, it was only a year after de Sitter gave his solution that Einstein, responding to criticism by Kretschmann (Kretschmann 1917), at last gave a formal definition of Mach's Principle (Einstein 1918):

> The G-field [the metric] is *completely* determined by the masses of the bodies. Since mass and energy are identical in accordance with the results of the special theory of relativity and the energy is described formally by the symmetric energy tensor ($T_{\mu\nu}$), this means that the G-field is conditioned and determined [*bedingt und bestimmt*] by the energy tensor of the matter.

However, despite asserting that he personally regarded its fulfillment as absolutely necessary, Einstein was finally forced to dissociate himself from Mach's Principle, though I do not think the yearning for it ever left him. In his "Autobiographical Notes" published in 1949 (Einstein 1949) he points out how mistaken he was on the subject, but the very next paragraph begins by saying that Mach's critique was in essence very sound, and Einstein illustrates his point by means of an analogy[8] that he originally was given by his close friend Michele Besso. It is interesting to note that Einstein first published the analogy in the dark days of 1914 when he felt obliged to work with noncovariant field equations but still had his irresistible hankering for general covariance. Incidentally, the journal in which it appeared was *Scientia*, published in Bologna (Einstein 1914).

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[8] Einstein's analogy was as follows:

> How sound, however, Mach's critique is in essence can be seen particularly clearly from the following analogy. Let us imagine people construct a mechanics, who know only a very small part of the Earth's surface and who also can not see any stars. They will be inclined to ascribe special physical attributes to the vertical dimension of space (direction of the acceleration of falling bodies) and, on the ground of such a conceptual basis, will offer reasons that the Earth is in most places horizontal. They might not permit themselves to be influenced by the argument that as concerns the geometrical properties space is isotrope and that it is therefore supposed to be unsatisfactory to postulate basic physical laws, according to which there is supposed to be a preferential direction; they will probably be inclined (analogously to Newton) to assert the absoluteness of the vertical, as proved by experience as something with which one simply would have to come to terms. The preference given to the vertical over all other spatial directions is precisely analogous to the preference given to inertial systems over other rigid co-ordination systems.

I quoted earlier a line from Keats' "Ode on a Grecian Urn." A few lines after the one quoted Keats gives what can be seen as a poetic summary of the whole saga of Einstein and Mach's Principle:

> Bold Lover, never, never canst thou kiss,
> Though winning near the goal – yet, do not grieve;
> She cannot fade, though thou hast not thy bliss,
> For ever wilt thou love, and she be fair!

What is most singular about the whole affair is that although in Einstein's view the passion never came to consummation two extraordinarily robust and handsome children were born: the general theory of relativity and modern relativistic cosmology.[9]

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[9] The present paper was not the occasion to put forward my own view on the matter, which is that Einstein did in fact create in his general theory of relativity a dynamical structure that is far more Machian (though in a sense rather different from what Einstein intended) than is generally believed (see Barbour and Bertotti 1982; Barbour 1989; Introduction; and Barbour 1990).

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