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Leibnizian time, Machian dynamics, and quantum gravity

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16.1. Introduction

Historically, dynamics was bedevilled from its beginning by the invisibility of space and time. Newton (1686) championed the view that space and time, although invisible, do exist and provide the arena within which motion occurs. Leibniz (1716) argued that there is no such thing as absolute space but only the relative configurations of simultaneously existing bodies and that time is merely the succession of such instantaneous configurations and not something that flows quite independently of the bodies in the universe and their motion. What Leibniz was advocating was that dynamics should be based exclusively on observable elements; it should not contain elements that are not in principle directly observable. This, of course, was Mach's standpoint too (Mach 1872), which Einstein (1916) adopted wholeheartedly when developing general relativity.

Quantum mechanics (QM) inherited the kinematic structure of Newtonian dynamics. Its most fundamental operators—those of momentum, angular momentum, and energy—correspond, respectively, to displacement and rotation in Newton's invisible space and displacement in invisible time. The aim of this paper is to question the extent to which the existing framework of QM is appropriate for quantization of Einstein's theory of general relativity (GR). It will be pointed out that the use of absolute space and time in Newtonian dynamics leads to a characteristic failure of predictive power. A simple theory of dynamics (gauge-invariant dynamics) will then be discussed that uses only directly observable quantities and does not suffer from this failure of predictive power. It will then be shown that GR has essentially the same structure as gauge-invariant dynamics. Finally, it is argued that for this reason it may not be appropriate to attempt to quantize GR within the existing framework of QM.

In his introduction to the discussion meeting, Roger Penrose pointed

out that hitherto the attempts to quantize GR have tended to regard QM as sacrosanct and have sought to make GR fit QM. He wondered whether, in view of the failure of this approach, the time had not come to look at the possibility of modifying QM. This paper is a contribution in that direction.

16.2. Predictive power of classical dynamics

The points raised in this paper can be illustrated by a *Gedanken-experiment*: suppose two successive 'snapshots' are taken of a universe of n material points (with known masses m_i, $i = 1, \ldots, n$) moving in Euclidean space under gravity in accordance with Newton's laws. The snapshots, which show only the relative distances between the bodies, differ intrinsically by a small amount, but the separation in time between them is not known. Is it possible to predict the future evolution of the system uniquely?

It is not. Some essential information is missing. From the relative distances in the two snapshots, it is impossible to deduce either the angular momentum or the kinetic energy of the system, but these two quantities exert a profound influence on the subsequent motion. Poincaré (1905) found such a failure of predictive power very curious. The equations of dynamics are of second order in time, so that initial positions and velocities are required in the initial-value problem. But in Newtonian dynamics, these quantities must be specified in absolute space and time. The purely relative—and observable—quantities do not quite suffice to determine the absolute quantities. As a result, quite different futures can evolve from apparently identical initial conditions (cf. Barbour 1982).

The next section presents a dynamical theory without this curious defect.

16.3. Gauge-invariant dynamics

Because the magnetic field is always observed to satisfy $\operatorname{div} \boldsymbol{B} = 0$, it can be represented in terms of a vector potential $\boldsymbol{A}$, $\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}$, with, however, $\boldsymbol{A}$ determined only up to the gauge transformation

$$\boldsymbol{A} \rightarrow \boldsymbol{A} + \nabla\psi, \qquad \psi = \psi(\boldsymbol{x}). \tag{16.1}$$

Similarly, the $n(n-1)/2$ observable relative distances between n ($n \geqq 5$) bodies in Euclidean space satisfy algebraic relations which reduce the number of independent quantities to $3n-6$. These relations make it possible to represent the observable data by means of Cartesian coordinates $\boldsymbol{r}_i$. Like the vector potential $\boldsymbol{A}$, these Cartesian 'potentials' are determined only up to the six-parameter 'gauge transformation'

$$r_i \to \mathscr{E} r_i + g, \tag{16.2}$$

where $\mathscr{E}$ is an orthogonal 3×3 matrix and g is a vector.

Imagine a continuous sequence of snapshots of the n bodies as they move relative to each other. Each snapshot represents an instant of Leibnizian time. Label the sequence by an arbitrary monotonically increasing label τ. Since the gauge of the potentials in each snapshot is independent of the gauges in the preceding and following snapshots, $\mathscr{E}$ and g in transformation (16.2) become arbitrary functions of τ:

$$r_i(\tau) \to \mathscr{E}(\tau) r_i(\tau) + g(\tau). \tag{16.3}$$

Note that the gauge group (16.3) is uniquely determined by the nature of the (algebraic) relations satisfied by the (observable) relative distances and the Leibnizian concept of time, according to which the label τ is itself determined only up to

$$\tau \to \tau', \quad \tau' = \tau'(\tau), \qquad d\tau'/d\tau > 0, \tag{16.4}$$

since no time label τ is distinguished *a priori*.

If the complete universe is considered, it is not difficult to construct a dynamical theory invariant under transformations (16.3) and (16.4), which may be called the *Leibniz group* (Barbour and Bertotti 1977). Only the essential features will be given here; the details can be found in Barbour and Bertotti (1982).

A gauge-invariant action is constructed. Let $\{r_i\}$ be an initial configuration and $\{r_i'\}$ differ from it infinitesimally: $r_i' = r_i + \delta r_i$. Now, if the gauge of, say, r_i' is changed, δr_i will change too. Let

$$\delta T = \sum_{i=1}^{n} m_i (\delta r_i / \delta\tau)^2, \tag{16.5}$$

and let $\delta \hat{T}$ be the minimum of eqn (16.5) over all δr_i obtained by applying transformation (16.3) to the representation of the second configuration. Then $\delta \hat{T}$ is uniquely determined by observable quantities and is gauge invariant.

To obtain the gauge-invariant analog of Newtonian gravitational dynamics, let

$$V = \sum_{i<j} m_i m_j / r_{ij},$$

and consider the variational principle

$$\delta I = 0, \qquad I = \int (V\hat{T})^{1/2} \, d\tau, \tag{16.6}$$

where the product form and the square root are taken in order to ensure that the integrand of eqn (16.6) is homogeneous of degree one and therefore invariant with respect to (16.4).

It is easy to show that the physically distinct solutions of eqn (16.6) are identical to the solutions of Newtonian gravitational dynamics for which the considered n-body universe has angular momentum about the centre of mass equal to zero and, in addition, zero total energy. These results are consequences of the invariance of eqn (16.6) with respect to transformations (16.3) and (16.4), respectively, or, as Bertotti and I put it, implementation of the First and Second Mach's Principles (Barbour and Bertotti 1982).

16.4. Two different concepts of motion

In the Newtonian concept of motion, each body moves primarily in the arena of absolute space and time, and only secondarily with respect to other bodies, the interactions with which give rise to deviations from the basic uniform rectilinear inertial motion. In the Leibnizian view, there is just a sequence of snapshots. Space and time are to be *constructed* from the observable data in the snapshots, which can be regarded as lying initially in a random heap.

Except in uninteresting degenerate cases, the presumed continuity of the changes of the relative configurations will permit a unique ordering of the sequence. After this, it is possible to *stack horizontally*. This is essentially the operation that occurs in the minimization of eqn (16.5). Take the first snapshot, place the second on top of it, and move the second around on the top of the first until eqn (16.5) is minimized. This will happen when: (1) the centres of mass coincide, (2) there is no overall rotation. This is what causes the total angular momentum to vanish. The third snapshot can then be stacked relative to the second, the fourth relative to the third, and so forth. The complete sequence is thus ordered and stacked horizontally.

To stack *vertically*, i.e., to determine a separation in 'time' between successive snapshots, we use the equations of motion deduced from eqn (16.6):

$$\frac{d}{d\tau}\left(\frac{V^{1/2}}{\hat{T}^{1/2}} \frac{dr_i}{d\tau}\right) = \frac{1}{2} \frac{\hat{T}^{1/2}}{V^{1/2}} \frac{\partial V}{\partial r_i} \tag{16.7}$$

(see Barbour and Bertotti (1982); the horizontal stacking is assumed already performed). Up to its origin, there is a unique choice of the label τ, which is still arbitrary in eqn (16.7), that casts eqn (16.7) into an especially simple form. Namely, choose τ such that

$$\hat{T}^{1/2} = V^{1/2}. \tag{16.8}$$

Then: (1) with respect to τ, the bodies now follow Newton's laws exactly. By virtue of the horizontal stacking, bodies subject to no forces move along straight lines; by virtue of the vertical stacking, they now move uniformly as well. As Poincaré (1905) asserted, 'time' is chosen so as to put the equations of dynamics into simplest form. (2) The determination of the distinguished time simultaneously defines an 'energy' of the system, which however is exactly zero (V is minus the usual Newtonian potential energy).

To summarize: to formulate gauge-invariant dynamics, it is not necessary to presuppose absolute space and time. Instead, they are constructed after the dynamical problem has been solved using observable relative data. It is necessary to consider the entire universe, which, after horizontal and vertical stacking, is necessarily found to have vanishing energy and angular momentum as a consequence of the gauge-invariant dynamics. However, well separated subsystems of the universe can perfectly well have non-vanishing values of these parameters. When applied to the complete universe, gauge-invariant dynamics is full rational in the sense required by Poincaré, i.e. seemingly idéntical initial data cannot give rise to quite different dynamical evolutions.

Kretschmann (1917) argued that Einstein's principle of general covariance has only methodological but not physical significance. For example, Newtonian dynamics can also be made invariant with respect to transformations (16.3) and (16.4) and, like gauge-invariant dynamics, be treated in a frame of reference in arbitrary motion. But this formal invariance—I shall call it *Kretschmann invariance*—is only possible at the price of the introduction of additional (and unobservable) elements. In contrast, the gauge invariance of Section 16.3 is achieved minimally. The Leibniz group (16.3) and (16.4) is unambiguously determined by the structure observed within each snapshot and by the concept of time. The minimal gauge invariance enhances the predictive power; Kretschmann invariance adds nothing.

16.5. General relativity as a generalization of gauge-invariant dynamics

Gauge-invariant dynamics solves the problem of determining the future from observable initial data. Of course, the type of 'snapshot', or 'simultaneity', can be generalized. For example, the snapshot might show the instantaneous values of a field or the distance relations that hold in a closed three-dimensional Riemannian space. Let us consider the latter case. Instead of the Cartesian 'potentials' $\mathbf{r}_i$, the 'potential' in this case is the metric tensor g_{ij}, $i, j = 1, 2, 3$, of the Riemannian space. The

analogue of the gauge group (16.3) is

$$g_{ij}(\tau) \rightarrow g_{ij}(\tau) + \Lambda_{(i;j)}(\tau), \tag{16.9}$$

where Λ_i are three arbitrary functions, the round brackets in the suffix denote symmetrization, and the semicolon denotes the covariant derivative with respect to g_{ij}. The Leibniz group for this problem is (16.9) and (16.4), and is not far short of being the general covariance group of four dimensions, i.e., the invariance group of general relativity. However, there is no mixing of (16.9) and (16.4).

The reason for (16.4) is that there is nothing intrinsic in any two snapshots to say how far apart they are 'in time'. But the division of the Leibniz group into (16.9) and (16.4) corresponds to the pre-relativistic notion that absolute simultaneity still has meaning. In the operational spirit of Section 16.4, this amounts to the assumption that the simplest form of the equations of motion can be obtained with a single time parameter, this time parameter being the same across the entire universe. In a post-relativistic approach, such a view cannot be maintained; one must consider the possibility that the separation in 'time' between the snapshots is not only unknown but also position dependent in general. Then (16.9) and (16.4) can no longer be kept separate and it is necessary to go over to the general covariance group in four dimensions.

Limiting the discussion to pure geometrodynamics (i.e. when no matter fields are present), the problem therefore is to find a generalization of the action (16.6). It was shown by Baierlein *et al.* (1962) that the action principle of GR can be put in the form

$$S[g_{ij}, N^k] = \int \mathrm{d}\tau \int \mathrm{d}^3x \{R[g] G^{ijkl}[g](\dot{g}_{ij} - 2N_{(i;j)})(\dot{g}_{kl} - 2N_{(k;l)})\}^{1/2}. \tag{16.10}$$

Here, R is the scalar curvature of the three-dimensional Riemannian spaces, $G^{ijkl} = \frac{1}{2}g(g^{il}g^{kj} + g^{ik}g^{lj} - 2g^{ij}g^{kl})$ is DeWitt's metric, $g = \det\|g_{ij}\|$, and N_i is the shift; $\dot{g}_{ij} = \mathrm{d}g_{ij}/\mathrm{d}\tau$; and indices are raised and lowered with g_{ij}. The summation convention is assumed.

The action (16.10) is a generalization of eqn (16.6). The variation with respect to the shift N_i (the 'thin-sandwich' problem described by Wheeler (1964)) is exactly analogous to the horizontal stacking described in Section 16.4. The shift enters eqn (16.10) in the way it does solely on account of (16.9). The really distinctive structure of GR dictated by the mixing of (16.9) and (16.4) into the four-dimensional covariance group is reflected by the appearance of DeWitt's metric G and the fact that R multiplies the remainder of the integrand at each point and there is only a single integral over space. Were invariance only with respect to (16.9) and (16.4) separately required, eqn (16.10) could be replaced by a much less specific expression.

Once the horizontal stacking has been performed, i.e., the thin-sandwich problem has been solved,† the vertical stacking is performed by *defining* a position-dependent lapse:

$$N = \tfrac{1}{2}R^{-1/2}(k_{ij}k^{ij} - k^2)^{1/2}, \tag{16.11}$$

where $k_{ij} = \mathrm{d}g_{ij}/\mathrm{d}\tau - 2N_{(i;j)}$. This is a position-dependent generalization of Poincaré's proposition that 'time' is chosen to make the laws of dynamics simple.

In gauge-invariant dynamics, the horizontal and vertical stacking of the snapshots leads to the construction of absolute space and time. In geometrodynamics, the 'heap' of three geometries is stacked analogously into a Ricci flat four-dimensional space. Moreover, in allowing arbitrary position-dependent 'time' separation between simultaneities, GR is a *non plus ultra* as regards predictive power. For this reason, one must agree with Wheeler (1964) that, at least in the case of a closed universe, GR is a faithful realization of Machian ideas.

16.6. Quantum mechanics and general relativity

As mentioned in Section 16.1, the attempts to quantize GR have been made under the assumption that QM has a basic structure with which one cannot tamper and to which GR must be matched. However, if we consider what features of Newtonian kinematics are essential to QM and then realize that gauge-invariant dynamics and GR are constructed in such a way as to eliminate precisely these features, we must question such an approach.

There are two features of Newtonian kinematics that play an essential role in QM: the phase space and the distinguished time variable. These are assumed to exist before any dynamics takes place in them. In the modern treatment of dynamics, for example, the phase space with its symplectic form is assumed given. Many different Hamiltonian evolutions can take place on one and the same phase space. In this view, the *momentum* of a particle exists prior to any dynamical law that it may satisfy. The basic structure of QM is set up prior to the dynamics, just as absolute space and time are assumed to exist prior to Newton's laws.

But this is not the case in gauge-invariant dynamics. It is only after the horizontal and vertical stacking (Section 16.4) that we recover the 'space–time' structure that corresponds to the arena provided by Newtonian absolute space and time. Moreover, the stacking procedure is

† Unfortunately, not much is known about the solution of the thin sandwich problem. Uniqueness has been proved under certain restrictions by Belasco and Ohanian (1969). This at least shows that the kind of non-uniqueness inherent in Newtonian dynamics is not encountered in GR.

carried out with one specific Lagrangian and the specific system (taken to represent the entire universe) one is considering. If divorced from these, a particle can be associated with neither a direction nor a magnitude of momentum.

We have seen that the kinematic structure of one-particle Newtonian dynamics may well have a Machian origin. It is a small step from this to the idea that some of the most basic features of QM have a Machian origin too. In particular, there is a suggestive correspondence between the failure of predictive power in Newtonian dynamics associated with the 'invisibility' of the energy and angular momentum of a system and the fact that wave packets in QM are constructed by superposition of eigenfunctions corresponding to different values of the quantum analogues of these very same quantities.

Smolin (see Chapter 10) has attempted to derive the one-particle Schrödinger equation in a many-particle global framework, but otherwise there seem to have been few attempts made in this direction.

16.7. The Hamiltonian constraints of general relativity

When GR was cast into Hamiltonian form as a preliminary to canonical quantization (Dirac 1958; Arnowitt *et al.* 1962), it was found that the dynamics of GR is not controlled by a conventional Hamiltonian but by so-called Hamiltonian constraints (Dirac (1964); see Kuchař (1981) for a modern review and an extensive bibliography). According to Kuchař, the presence of these constraints 'incredibly complicates the implementation of the canonical quantization programme'. In this connection, it is interesting to examine the literature to see what attitude the investigators took to the problem of dealing with these constraints. Overall, the tendency seems to have been to look for similar constraints in systems that have already been successfully quantized and hope that these would give guidance in the case of GR. However, the constraints with which comparisons were made arise from what I have called Kretschmann invariance (Section 16.4), i.e., a formal invariance achieved by adding to the original dynamical elements of the theory without in any way increasing the predictive power of the theory.

This is well demonstrated by the so-called super-Hamiltonian constraint, which was claimed to be analogous to the constraint that arises in parametrized particle dynamics (Dirac 1964; Lanczos 1949). This similarity was taken as a guide to the way in which one should attempt to identify the time variable in the quantization of GR, the point being that in parametrized particle dynamics 'time' is included among the ordinary dynamical variables, and it was concluded that in GR 'time' is hidden among the dynamical variables, which in the Hamiltonian formulation are the components of the spatial metric g_{ij}, $i, j = 1, 2, 3$, and the momenta conjugate to them. However, I want to argue here that the super-Hamiltonian constraint of GR is rather an indication that there is no 'time' at all in GR. To do this, it will be necessary to consider two different constraints: the one that arises in parametrized particle dynamics and derives from a Kretschmann invariance and the one that arises in gauge-invariant dynamics and derives from a minimal invariance.

Let $\mathscr{L}(q_i, \dot{q}_i, t)$, $i = 1, \ldots, 3n$, be the Lagrange function of n Newtonian point particles with kinetic energy T and potential energy U. Parametrize the particle paths by an arbitrary time label τ ($\mathrm{d}q_i/\mathrm{d}\tau = q_i'$) and adjoin the absolute time $t = q_0$ to the q_i's. The new Lagrangian $\bar{\mathscr{L}} = \bar{\mathscr{L}}(q_i, q_i', t, t')$ is homogeneous of degree one in the 'velocities' q_i' and t', so that the action is invariant with respect to (16.4). It is well known that as a result the Hamiltonian corresponding to $\bar{\mathscr{L}}$ vanishes identically (Dirac 1964). To obtain a Hamiltonian description, it is necessary to introduce a constraint. Namely, the Hamiltonian description of the parametrized form of the dynamics is derived from the principle

$$\delta I = 0, \quad I = \sum_{i=0}^{3n} \int [\pi_i q_i' - N(\pi_0 + H)]\, \mathrm{d}\tau, \qquad (16.12)$$

where H is the Hamiltonian corresponding to $\mathscr{L}$ (the 'physical' Hamiltonian) and $\pi_i = \partial\bar{\mathscr{L}}/\partial q_i'$. Variation with respect to the Lagrange multiplier N gives the constraint

$$\mathscr{H} \equiv \pi_0 + H = 0. \qquad (16.13)$$

Variation with respect to π_0 gives the 'lapse'

$$N = t', \qquad (16.14)$$

which remains an arbitrary function not determined by eqn (16.12). This is what is known as the parametrized form of dynamics. In it, the absolute time t and $-H$ appear as an extra pair of canonical co-ordinates. The parameter invariance of eqn (16.12) is a typical Kretschmann invariance, achieved by adding t and $-H$ to the dynamical variables. The predictive power of the theory is unchanged.

Now suppose the system is conservative, i.e. $\mathscr{L}$ does not contain t explicitly. Then t is an ignorable co-ordinate (see Lanczos 1949) and can be eliminated by the Routhian procedure, giving Jacobi's principle for the path in configuration space:

$$\delta J = 0, \qquad J = \int (E - U)^{1/2} s'\, \mathrm{d}\tau, \qquad \mathrm{d}s^2 = \sum_{i=1}^{3n} m_i\, \mathrm{d}q_i\, \mathrm{d}q_i, \quad (16.15)$$

where E is the constant total energy: $E = T + U$. Thus, in eqn (16.15), E

is regarded as a constant, but U is configuration dependent. In Newtonian dynamics, the *speed* at which the system moves through its configuration space is found from the energy equation by quadrature after the 'orbit' problem (16.15) has been solved.

Note that the principle (16.12) yields solutions of the original problem with *all* values of the energy, but eqn (16.15) yields only those with the value E. In particular, if $E = 0$ we recover our result of Section 16.3. Let us assume this is the case. Now the variational principle (16.15) is still invariant under (16.4), so that the Hamiltonian corresponding to eqn (16.15) again vanishes. The only way we can obtain a Hamiltonian description now is to take the action

$$I_0 = \sum_{i=1}^{3n} \int (\pi_i q_i' - N_0 H)\, \mathrm{d}\tau. \qquad (16.12')$$

Variation with respect to N_0 gives

$$H = 0, \qquad (16.13')$$

and ~~consistency enforces~~ one can require

$$N_0 = 1. \qquad (16.14')$$

So far as I know, a super-Hamiltonian constraint like eqn (16.13') has not hitherto been considered in the literature, though it seems to be more appropriate for comparison with the constraint in GR. The difference between eqns (16.13) and (16.13') is made clear by the two-snapshot initial-value problem considered in Section 16.2 and the distinction made in Section 16.4 between Kretschmann invariance and minimal invariance. The variables which occur in eqn (16.13) are *heterogeneous*. Namely, the momenta corresponding to the ordinary dynamical variables are all directly observable (we can assume for the sake of the discussion that the horizontal stacking has already been performed) whereas the momentum corresponding to the time variable ($-H$) is not observable. For given values of the remaining (observable) dynamical variables, the numerical value of H is undetermined. This is true in both the conservative and the non-conservative case, and it corresponds to the one-parameter uncertainty in the dynamical evolution from observable initial conditions noted in Section 2. In contrast, in eqn (16.13') there is no such time variable. All the variables are observable, and the future is uniquely determined by the observable data. Since GR shares this property with gauge-invariant dynamics, one must consider the possibility that the attempt to find a 'time' variable in GR is misconceived.

As an alternative, one could at least look at the quantization of the simplest problem in gauge-invariant dynamics that has constraints of the same intrinsic structure as GR: the three-body problem of celestial dynamics in the centre-of-mass frame with vanishing total angular momentum (analogue of the super-momentum constraint) and zero total energy (super-Hamiltonian constraint). In particular, if a 'time' is to be found among the ordinary dynamical variables, it will have to be constructed from among the genuine observables of the system, i.e. the three relative distances between the bodies (thus, 'time' could be either the perimeter or the area of the triangle formed by them). This makes it clear that quantization of GR differs in a fundamental way from the quantization of all other systems hitherto considered, a fact that has perhaps been obscured rather than illuminated by formal Kretschmann-type analogies.

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17

Quantum time–space and gravity

David Finkelstein and Ernesto Rodriguez

17.1. Introduction

It is far-fetched to describe quantum gravity by a quantum (q) pseudo-metric tensor g depending on classical (c) coordinates x, a theory we call here cq for its hybrid classical-quantum nature. After all, consider how we determine these variables.

We may determine coordinates of an event by four explosions which hurl good clocks in all possible directions at all possible speeds. The four readings on the four clocks that reach an event are its coordinates.

We may determine the interval between events by setting off a similar explosion at one event; the reading on the clock that reaches the other event is the interval.

It would be arbitrary to say that the four variables resulting from the first procedure are c-numbers, while that from the second would be a q-number. The cq theory does not go so far; it claims that because the shrapnel consists of clocks, it acquires a quantum indeterminacy from the gravitational field. Were the shrapnel charged, it would pick up the quantum nature of the electromagnetic field it traverses as well. To determine c coordinates, some reliable and continuous number-generators insensitive to gravity and any other q field must replace the clocks. This still seems far-fetched, in view of the universal coupling to gravity. Gravity determines propagation and the characteristic surfaces of all other fields.

It seems more likely that gravity and its light cones reflect a fundamental microscopic structure of time space, and that the q nature of gravity derives from that of this microstructure.

In the c theory, the co-ordinates belong to the level of the manifold, the third level in the hierarchy of theories of Table 17.1. The usual (cq) theory of fields, applied to gravity, assumes that the first three levels are c and the last three are q. We have been exploring a physics which is q all the way down, with a q correspondent for each of these levels. We report (Sections 17.2 and 17.3) on the present state of our q gravity. One problem we all have is giving meaning to a psi vector for the universe. We explain (Section 17.4) how and why we do not do this.