Entropy and the Universe

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JB, T. Koslowski, and F. Mercati,“Identification of a gravitational arrow of time”,
Phys. Rev. Lett. **113**, 181101 (2014),
“Entropy and the typicality of universes”, arXiv: 1507.06498v2.
Diagrams by Flavio Mercati.

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Two Problems

1. Given **time-reversal symmetric** laws,

whence come the **arrows of time?**

- Growth of entropy
- Growth of structure and information
- Retarded potentials

2. **Do we live in a typical universe**?

Key Insight

Conventional thermodynamic systems are **confined**.

The universe is **unconfined**

Arrow-of-time literature reveals little awareness of the difference.
The two cases require **very different conceptualization**.

We identify **two** entropy-like quantities in the universe:
Decreasing Entaxy and **Increasing Entropy**

Summary

A realistic proof-of-principle N-body model suggests:

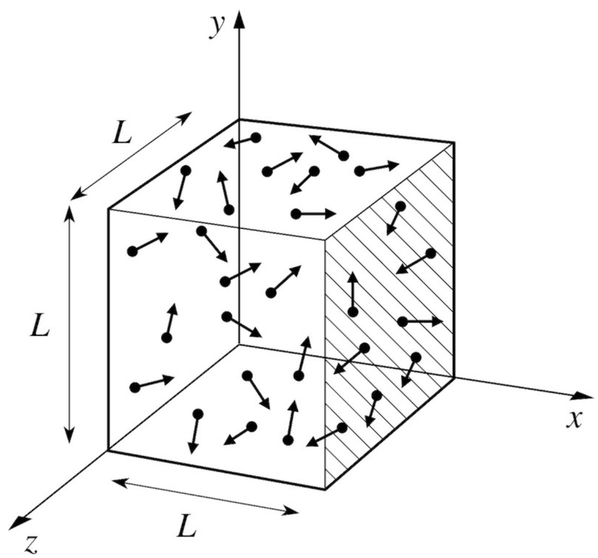
1. The Second Law of Thermodynamics must hold in a self-gravitating universe. No special ‘initial condition’ required.

2. If the law that governs the Universe is known, it will have typical solutions about which strong predictions can be made.

3. These predictions appear to hold in our Universe.

Meaning and Significance of Confined

Ideal gas in a box



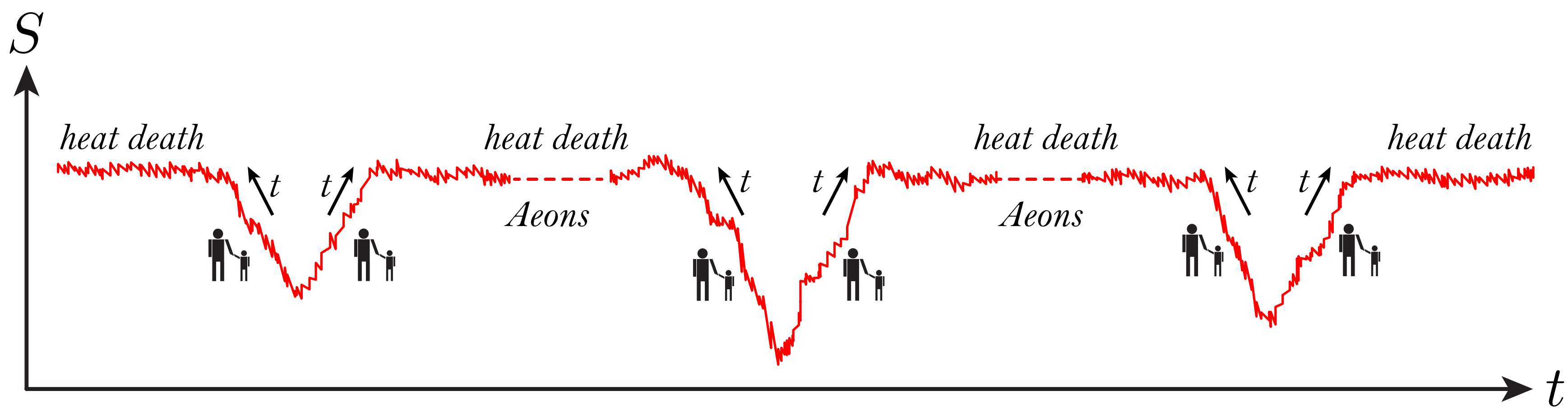
Experimentally box permits controlled supply and extraction of heat; measurement of pressure P, volume V and temperature T in equilibrium; and thus determination of entropy:

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} + \frac{P\mathrm{d}V}{T}$$

Theory: Gibbs (1902) created statistical mechanics for general Hamiltonian systems but **to count microstates** ($S = k \log W$) imposed conceptual boxes: **bounded measures** of (1) configuration space, (2) momentum space (no $1/r^2$ forces $\Rightarrow$ no gravity. Both ‘unnatural’).

Consequences of Confining Box

Free expansion ‘thwarted’ $\Rightarrow$ equilibrium and infinitely many Poincaré recurrrences. Solutions are **qualitatively time-reversal symmetric**.



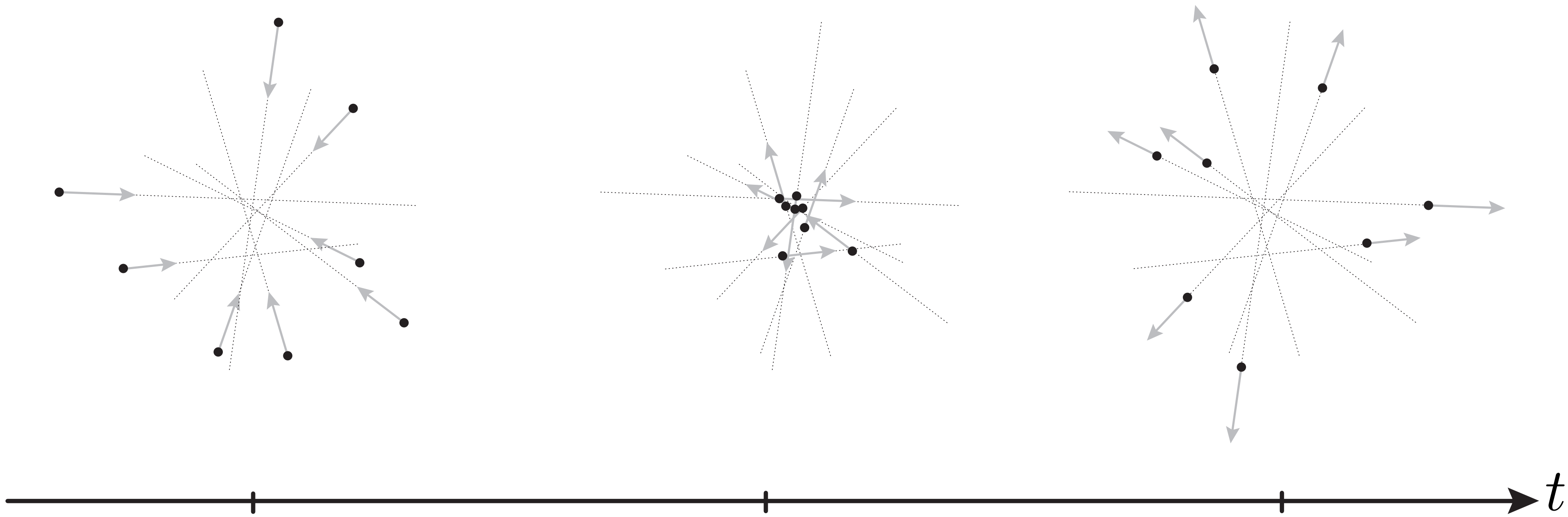
Boltzmann: “The universe is, and rests forever, in thermal equilibrium.” Only near deep entropy dips “are worlds where visible motion and life exist . . . the direction of time towards the more improbable state [will be called] the past.” This is a **one-past–two-futures** interpretation of each improbable state.

Boltzmann: Second Law is due to huge fluctuation in our remote past.
Today: Big Bang must have had an exceptionally low entropy.

Unconfined Janus-Point Systems

Every solution splits in two at a unique **Janus point J**. Either side of **J** evolution is **time-asymmetric**. Each solution has **one past and two futures.** Observers in either half must find an **arrow of time**. All solutions similar: **no special initial conditions**.

1. Simplest Janus System: Inertial Motion



$$\boxed{\mathbf{r}_a(t) = \mathbf{r}_a(0) + \mathbf{v}_a(0)\, t}$$

Moment of inertia $I_{\text{cm}} = \sum_a m_a \, \mathbf{r}_a^{\text{cm}} \cdot \mathbf{r}_a^{\text{cm}}$ has **one minimum** (**Janus point J**). Away from J system tends to Hubble-type expansion ($\dot{r}_{ab} \propto r_{ab}$), so positions and momenta **weakly correlated at J** but **highly correlated** as $t \to \pm\infty$. Does not look like entropic decay of order into disorder.

2. Dissipationless Waves of Compact Support

Disorder $\rightarrow$ Outgoing Retarded Waves

Time reversal (gauge transformation). Then

'Fine-Tuned' Incoming Waves $\rightarrow$ Disorder

Einstein (1909): Advanced waves not observed because 'initial conditions' statistically improbable. But

Outgoing Waves $\leftarrow$ Disorder $\rightarrow$ Outgoing Waves

suggests that we live in a Janus-type universe in which retarded potentials are **'attractors'** in both asymptotic regions.

Is the Universe Confined or Unconfined?

....

Disorder $\leftarrow$ **Order** $\rightarrow$ **Disorder**
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Disorder $\leftarrow$ **Order** $\rightarrow$ **Disorder**

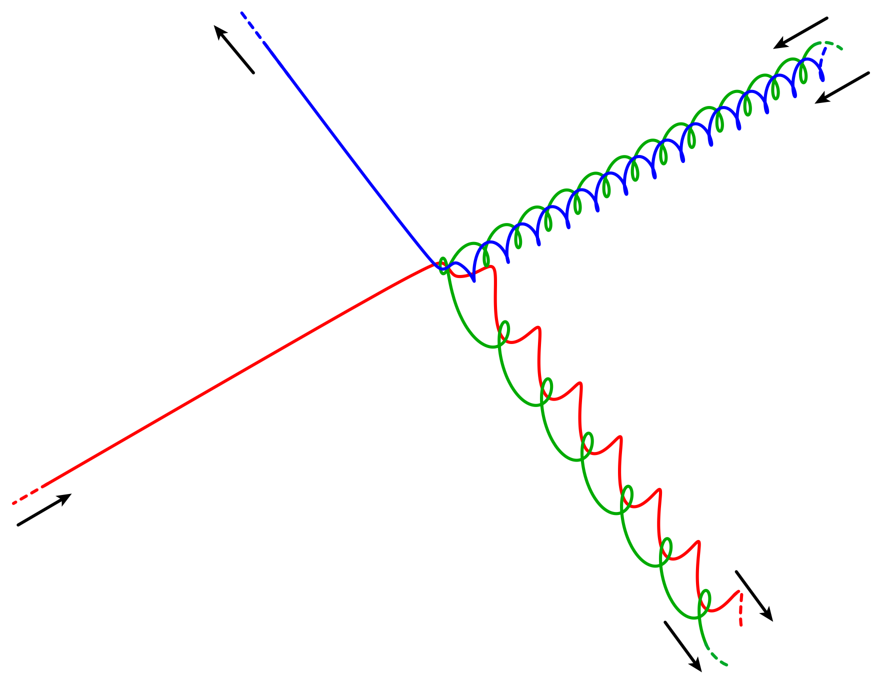
Order $\leftarrow$ **Disorder** $\rightarrow$ **Order**

Poincaré: **time direction** (statistical) is from **Order** to **Chaos**

Janus: **time direction** (dynamical) is from **Chaos** to **Order**

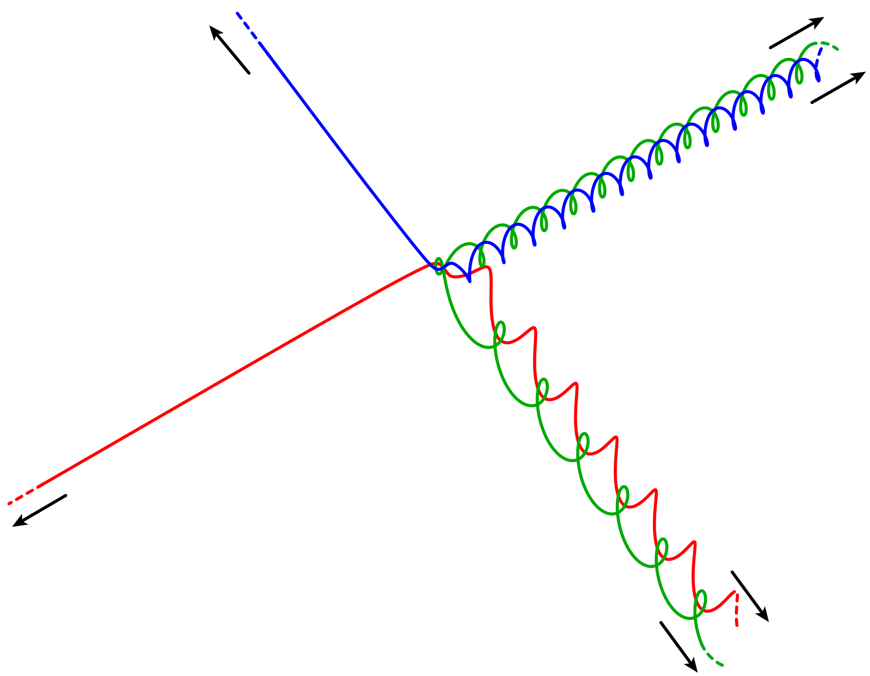
3. The Relational N-Body Problem

Gravity makes things much more realistic and interesting.
Relational: The conditions $E_{\text{cm}} = \mathbf{L} = 0$ exclude absolute elements.
All key effects present in the simplest non-trivial case: **the 3-body problem.**



Hyperbolic–Elliptic Escape: Singleton from left meets Kepler pair in 3-body interaction $\rightarrow$ (new or old) singleton and pair.

One-Past–Two-Futures Interpretation

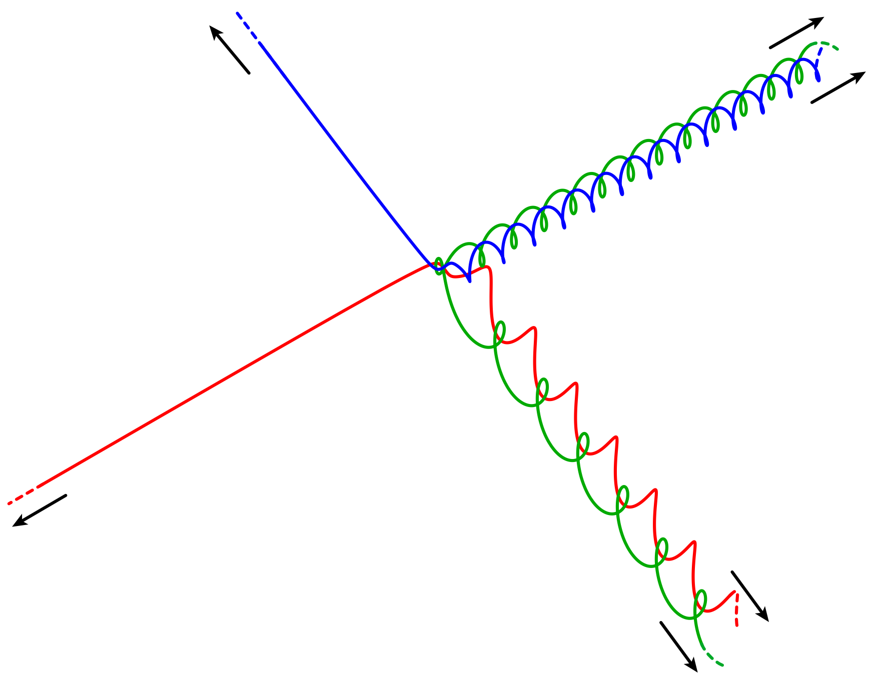


Each diagonal is a history that begins in the central Janus-point region from which pair and singleton emerge. Qualitative symmetry of histories matches exact symmetry of equation. Time from ‘big-bang’ measured by periods of emergent Kepler pair.

In each history, the ‘universe’ **expands** and breaks up into **branch systems**.

Gravity creates **structure out of chaos.** For internal observers, **only sensible choice for direction of time** fixed by growth of structure, information and records (orbital elements stabilize ever better).

Noether II Universe, Noether I Branch Systems



The relational N-body universe is a Noether-II gauge system **with constraints** $E_{\text{cm}} = \mathbf{L} = 0$. But the branch systems asymptote to Noether-I systems with conserved non-vanishing energy E, momentum $\mathbf{P}$ and angular momentum $\mathbf{L}$. **Treat differently!**

For the N-body Universe, we introduce the entropy-like quantity **entaxy**, which **decreases** in both directions away from the Janus point J.

We show that branch systems are born with a **Boltzmann entropy** that **increases** in both directions away from J.

Statistical Mechanics for Universes

Counterparts of Gibbs' Conditions:

- **Any entropy-like quantity** must be a **count of microstates** in a phase-space region of **bounded-measure** defined by **a state function**. Mere monotonic increase does not make an entropy.

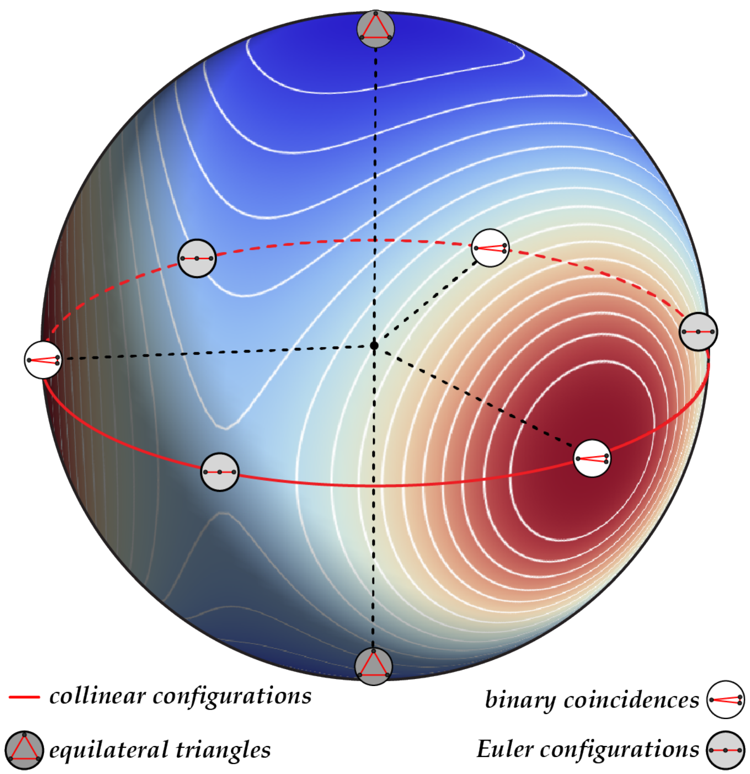
- **State functions and measures** must be **scale-invariant** (no rods and clocks external to the Universe).

- State functions should reflect **fundamental properties of the Universe** (like energy in conventional stat mech to find $S(E)$).

Our Aim: To determine **the typicality of universes**
(Like Gibbons–Hawking–Stewart (1986) but with significant differences).

The Key Scale-Invariant Concept: Shape Space S

The N-body Newtonian configuration space Q has 3N degrees of freedom. Translations and rotations are 3 + 3. The (centre-of-mass) moment of inertia $I_{\text{cm}} = \sum_{a<b} m_a m_b \|\mathbf{r}_a - \mathbf{r}_b\|^2$ is the scale dof. The remaining 3N-7 shape dofs (and mass ratios) define **Shape Space S** (quotient of Q wrt the similarity group). Shape space is **compact with bounded measure** (no box needed).



If N = 3 shape space is the space of triangle shapes. Since 2 internal angles fix a triangle, S corresponds to 2D **Shape Sphere** shown here for the equal-mass case. Points at equal longitude but opposite latitudes represent mirror-image triangles. The degenerate collinear triangles lie on the equator, the equilateral triangles at the poles. Colour coding to be explained.

Newton in Shape Space

Newtonian solutions project to undirected unparametrized curves in S. If they were geodesic, **a point and direction** would fix a solution. Laplacian shape determinism fails:

- Absolute orientations give 3 components of angular momentum $\mathbf{L}$.
- Absolute (metric) time allows different values of the energy E.
- Absolute scale allows moment of inertia I_{cm} to change.

Cauchy data in S: shape, direction and 3 + 1 + 1 numbers. Relationism enforces $E = \mathbf{L} = 0$ but scale (I_{cm}) remains.

Cauchy data are **Shape + Tangent Vector**, not Shape + Direction.

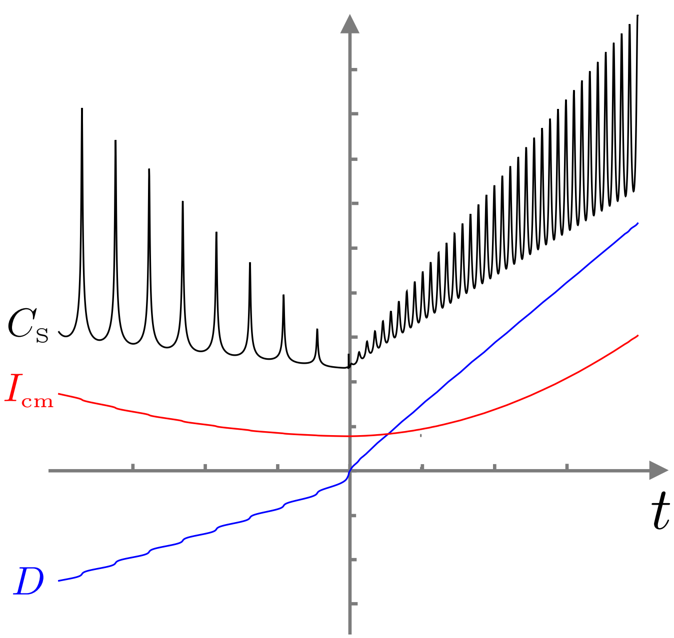
Single extra Hamiltonian dof is architectonic

The Lagrange–Jacobi (Virial) Relation

If V homogeneous, $\boxed{V(\alpha \mathbf{r}_a) = \alpha^k V(\mathbf{r}_a)}$, then $\boxed{\frac{1}{2}\ddot{I}_{\text{cm}} = E_{\text{cm}} - 2(k+2)V}$

If $\boxed{E_{\text{cm}} \geq 0}$, then because $\boxed{V_{\text{New}} < 0}$ and $\boxed{k = -1}$ we have $\boxed{\ddot{I}_{\text{cm}} > 0}$

The moment of inertia is U-shaped upwards, the **dilatational momentum** $D = \sum_a \mathbf{r}_a \cdot \mathbf{p}^a \ \ (= \frac{1}{2}\dot{I}_{\text{cm}})$ is **monotonic** and **vanishes once** (figure).



Architectonics: D is a Lyapunov variable $\Rightarrow$ no periodic solutions $\Rightarrow$ **no Poincaré recurrences**; at Janus point $D = 0$ **a point and direction** do determine a solution. The relational N-body problem is not quite scale-invariant, but its Cauchy problem is. **Mid-point solution-determining data** at $D = 0$ are **scale-invariant, non-redundant and unbiased.**

Dynamical Similarity

If V homogeneous, then

$$\boxed{\mathbf{r}_a \rightarrow \alpha \mathbf{r}_a, \quad t \rightarrow \alpha^{1-k/2} t,}$$

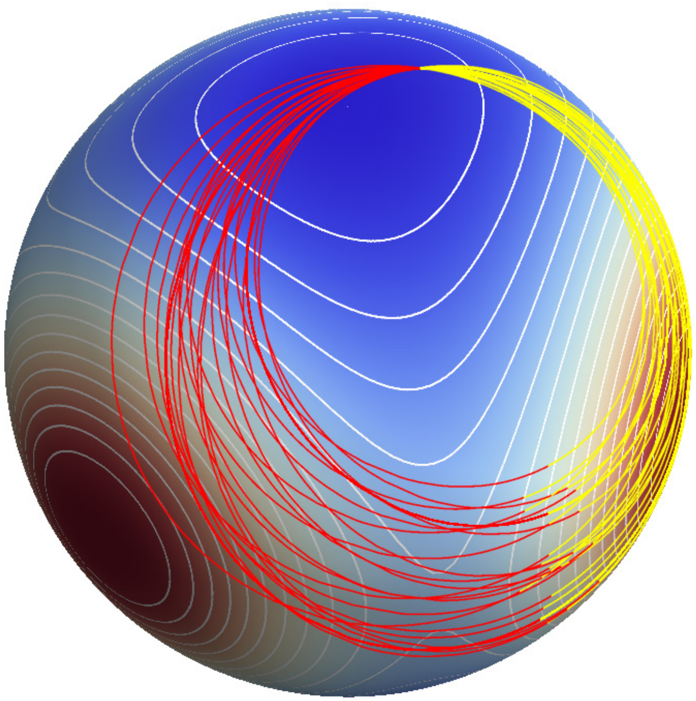
maps solutions to geometrically similar solutions ($k = -1$ generalizes Kepler's Third Law). NB: not a Noether symmetry.

Solutions with initial momenta of same direction but different magnitudes are identical in Shape Space. Gibbs' exclusion of $1/r^2$ potentials not needed.

Since S is compact, both Gibbs 'boxes' for confined systems **arise naturally for true observables of the unconfined N-body universe**.

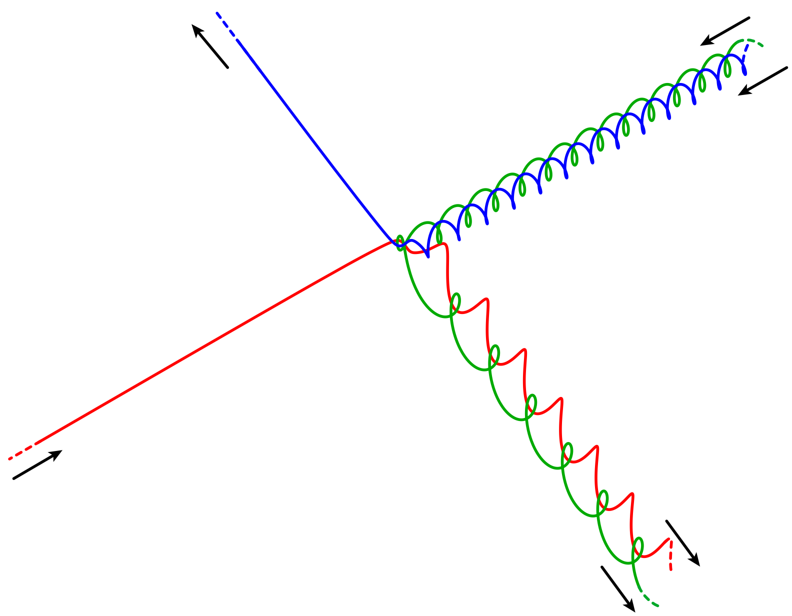
Attractors in Shape Space

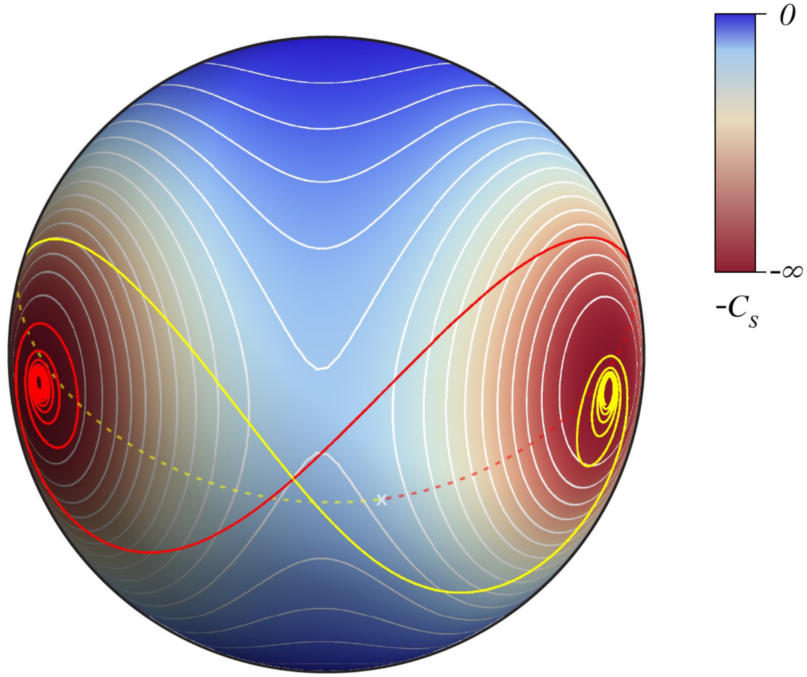
Liouville measure is conserved in the Newtonian phase space. Therefore, **if scale part increases** $\Rightarrow$ **shape part decreases**. In inertial motion, $\boxed{\mathbf{r}_a(t) = \mathbf{r}_a(0) + \mathbf{v}_a(0)\, t}$, so $\mathbf{r}_a$ tend as $t \to \pm\infty$ to a shape fixed by $\mathbf{v}_a(0)$.



Gibbs ensemble of 20 two-dimensional inertial motions with identical velocities but different initial positions. In shape space the ensemble contracts as $t \to \pm\infty$ to a single common shape.

Attractors in Newtonian Gravity





For dynamics of shapes $\boxed{-\log C_\mathsf{S}}$ acts as a shape potential and the dynamics appears dissipative. **Attractors + gravity $\Rightarrow$ isolated clusters form $\Rightarrow$ emergent Second Law in all solutions.**

A Scale-Invariant Measure of Clustering

Expansion of universe is not 'seen' but deduced from evolution of ratios:

$$\frac{\text{Galactic Diameters}}{\text{Inter-Galactic Separations}} \to 0$$

To reflect this, we define a scale-invariant measure of clustering C_{S} as the ratio of two mass-weighted lengths: the **root-mean-square length** ℓ_{rms} and the **mean harmonic length** ℓ_{mhl}

$$\ell_{\mathsf{rms}} := \sqrt{\sum_{a<b} \frac{m_a m_b r_{ab}^2}{m_{\mathsf{tot}}^2}} = \sqrt{I_{\mathsf{cm}}/m_{\mathsf{tot}}},$$

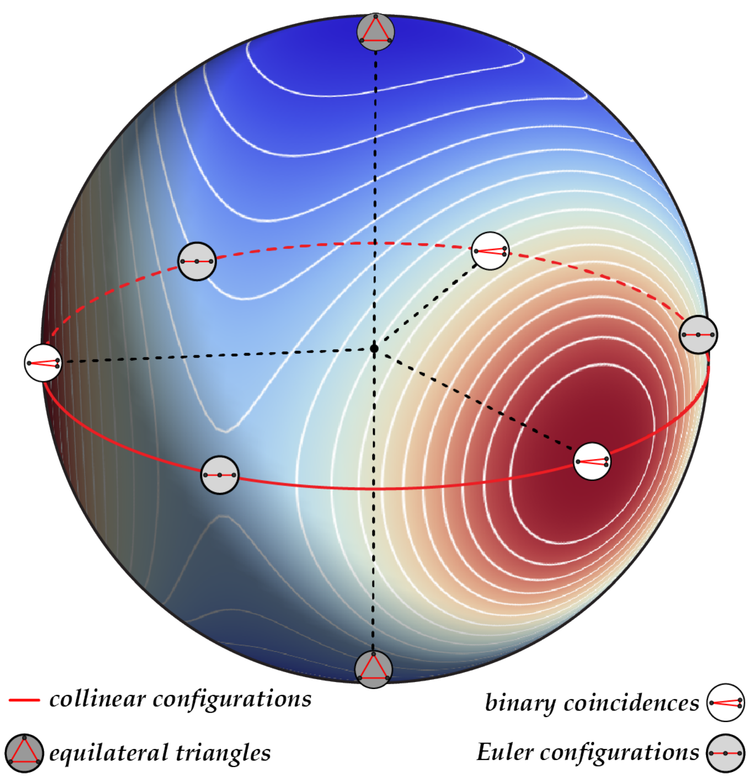
$$\ell_{\mathsf{mhl}}^{-1} = \frac{1}{m_{\mathsf{tot}}^2} \sum_{a<b} \frac{m_a m_b}{r_{ab}} = \frac{1}{m_{\mathsf{tot}}^2} V_{\mathsf{New}},$$

Shape Complexity $C_{\mathsf{S}} = \ell_{\mathsf{rms}}/\ell_{\mathsf{mhl}}$.

A sensitive measure of clustering

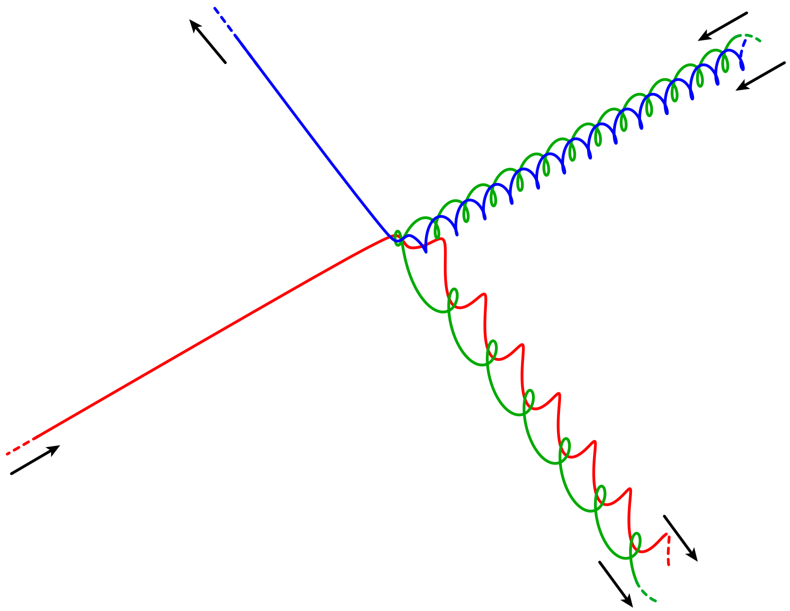
Shape Complexity as a State Function

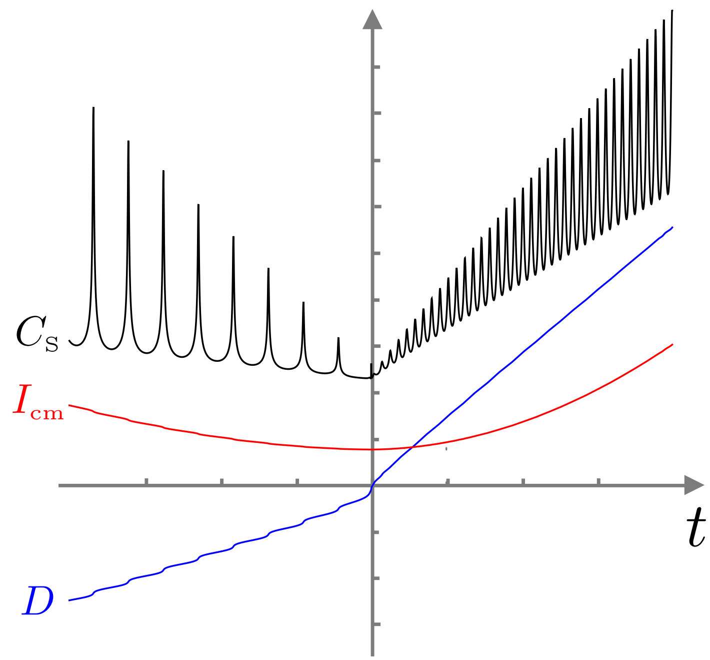
We cannot use E_{cm} as state function (it is zero). However, C_{S} is epoch dependent and, like E, dynamically fundamental ($-C_{\text{S}}$ is the **shape potential**). (Newtonian gravity seems 'designed' to create structure!)



Colour coding shows C_{S} (minima at poles, infinite at 'needles'). All **shape microstates** on a C_{S} contour belong to a C_{S}-**macrostate**. We will define a metric on S that makes it possible **to count microstates and define an entropy-like quantity for the universe**. Besides getting an emergent Second Law in all solutions, we can **calibrate the typicality of every solution.**

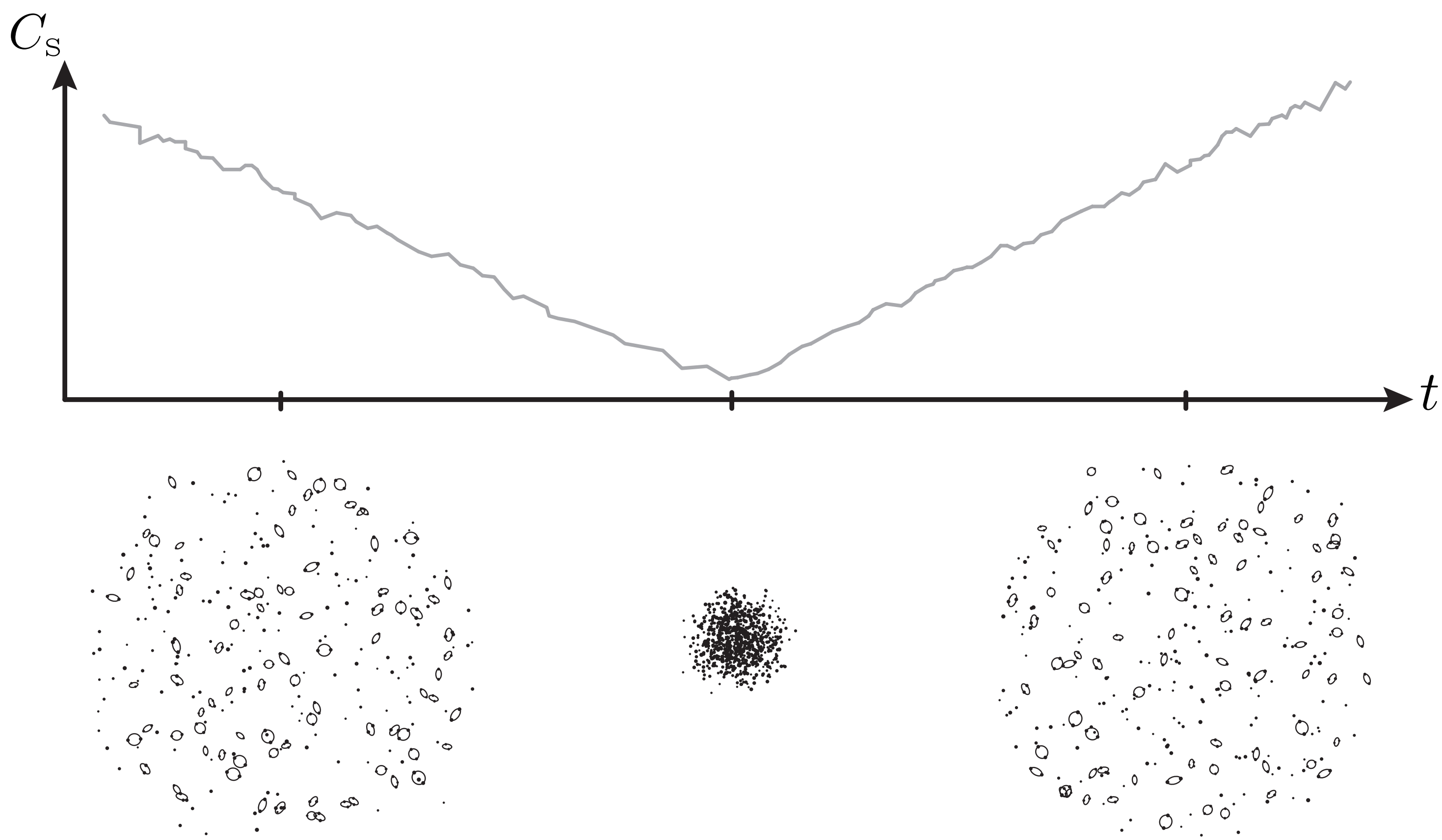
Typical 3-Body Solution





Creation of complexity, information and physical rods and clocks with respect to which the universe ‘expands’.

1000-Body Simulation



Sumulation Above: large N smooths wiggles. Below: ‘Artistic impression’.

Mid-Point Data

Each solution has its unique Janus point J.
At J, a **shape and direction** in S determine a solution.
At J, set **scale-invariant, unbiased, non-redundant mid-point data**.
Minimal encoding of all objective information conserved by evolution.

Elements of cotangent bundle T^*S of S have direction and magnitude
but magnitudes irrelevant (dynamical similarity) $\Rightarrow$ points of PT^*S
projectivized cotangent bundle (Janus manifold) fix distinct solutions.

PT^*S is odd-dimensional (contact geometry, Arnold's *Mechanics*),
and its points map **one-to-one onto** the solution space.

Measure on the Solution Space

A symplectic measure is induced on T*S (Arnold) but is infinite (momentum magnitudes unbounded, Gibbs $1/r^2$ restriction).

On Q there is a unique unbiased scale-invariant mass metric:

$$ds^2 = \sum_{a=1}^{N} \frac{m_a \, d\mathbf{r}_a \cdot d\mathbf{r}_a}{I_{\text{cm}}}.$$

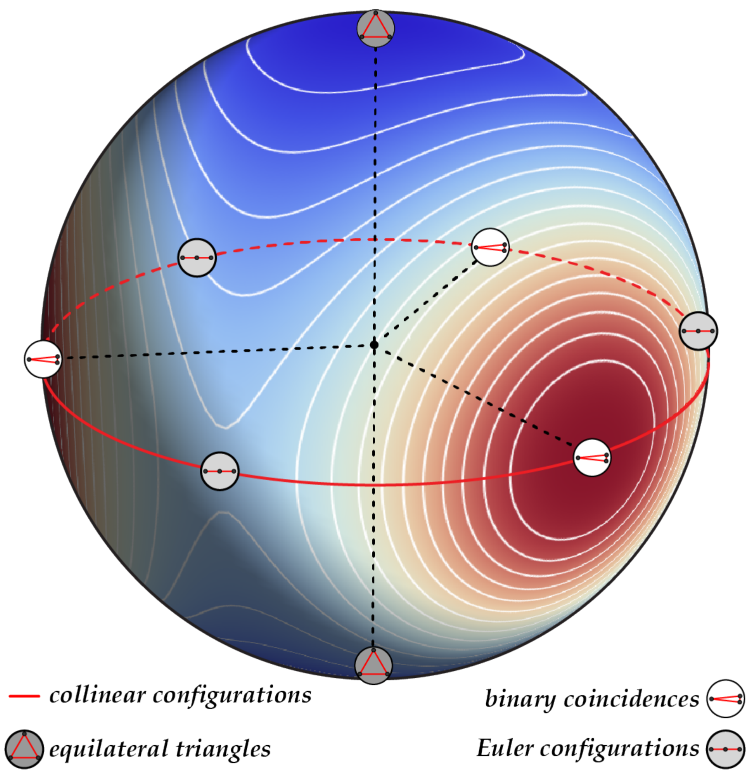
Riemannian quotienting induces a distinguished metric on S.

At any shape in S, the metric above makes it possible to attribute a norm to shape momenta.

Shape Space and (Relative) Shape Momenta have **bounded measures** without Gibbs-type restrictions.

Typicality of Solutions: Entaxy

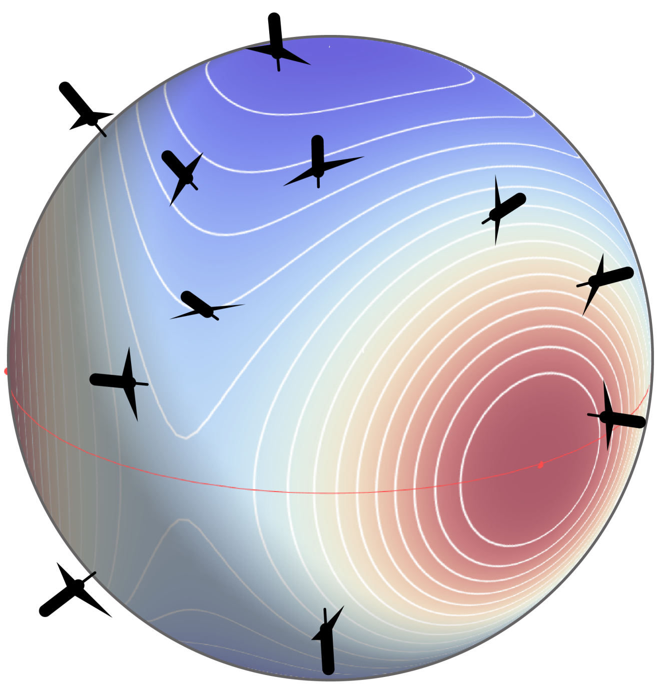
Use C_{S} and a momentum analogue C_{M} as **state functions**.



All **shape microstates** on a C_{S}, C_{M} level surface belong to a **shape macrostate** with volume $\mathcal{E}_{\mathrm{S,M}}$ induced by the shape metric. We call $\mathcal{E} = \log \mathcal{E}_{\mathrm{S,M}}$ the **entaxy** of a solution that has complexities C_{S}, C_{M} at its Janus point. Entaxy is a scale-invariant entropy-type quantity: **how likely it is for a universe to have a given shape and shape-space direction at its Janus point.** Effective initial data for internal observers.

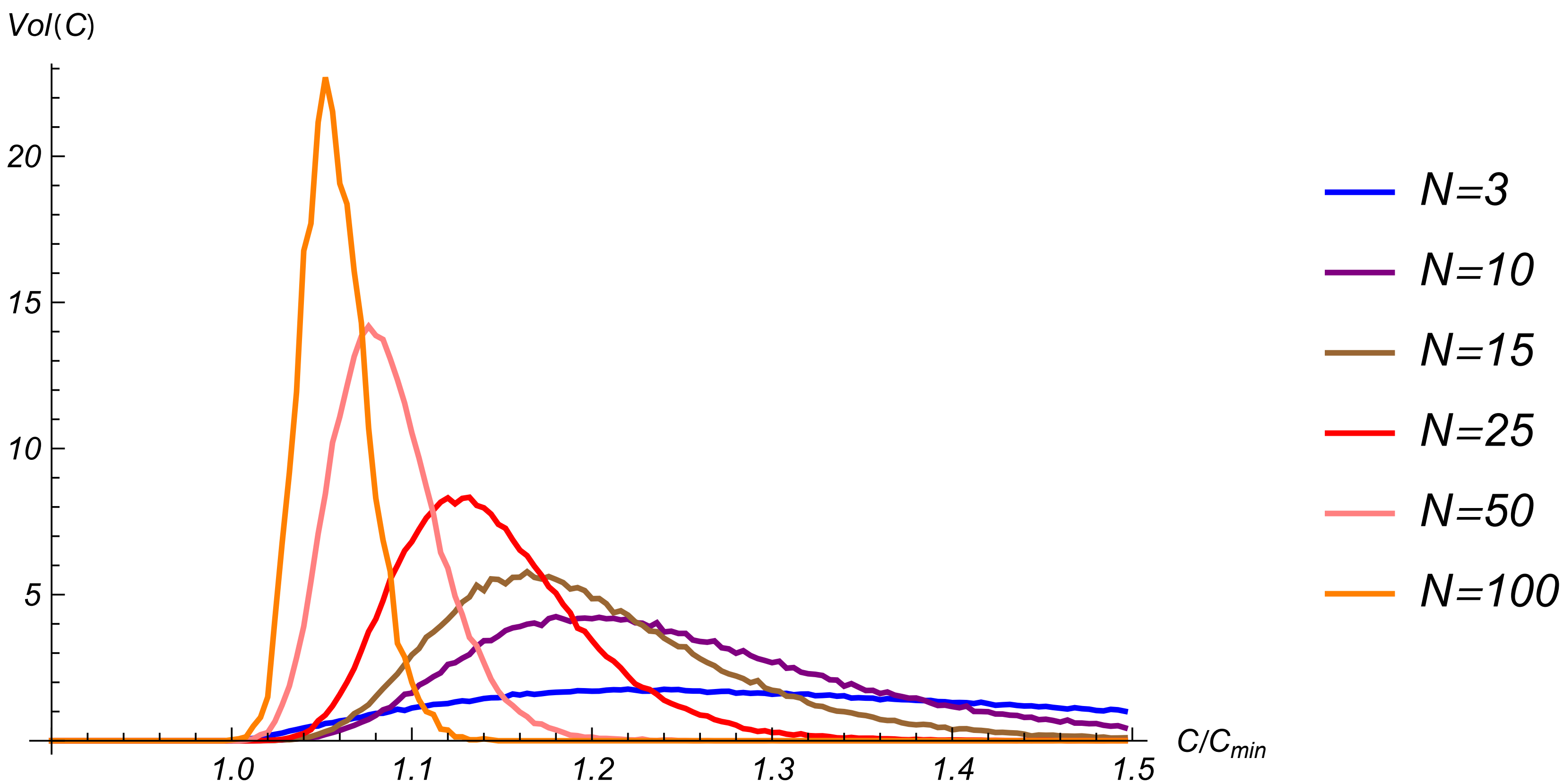
The Known Unknown: In Which Solution Are We?

We know the law of the universe. Can we predict what the **typical solutions** will be like? **Laplace's Principle of Indifference:** if a certain number of outcomes are possible but nothing else known, **give equal probability to each** (maximum entropy principle, fundamental postulate of stat mech).



Like GHS, but now with bounded measure and total control of solution space, we follow Laplace and give **equal probability to equal-measure regions.** The probability of occurrence of a universe is proportional to **its entaxy at its Janus point.** The probabilities with which triangle shapes occur at the Janus point can be read off from the figure (few 'needles').

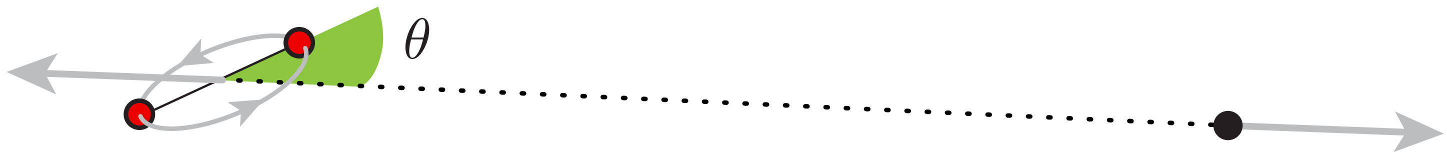
Shape-Space Volume vs. Complexity

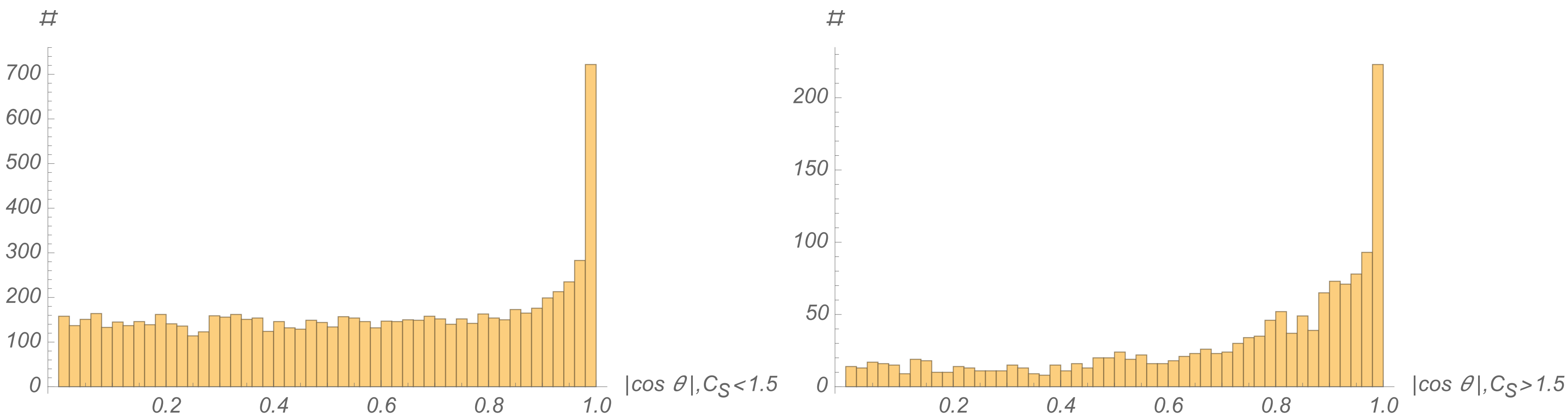


Measure of shape macrostates as function of $C_{\mathrm{S}}/C_{\mathrm{min}}$. Sampling caveat but suggests **typical universes will be very uniform** at the Janus point.

With one exception, we avoid Schiffrin–Wald objections to GHS.

Late-Time θ vs. Janus-Point C_S in 3-Body Problem

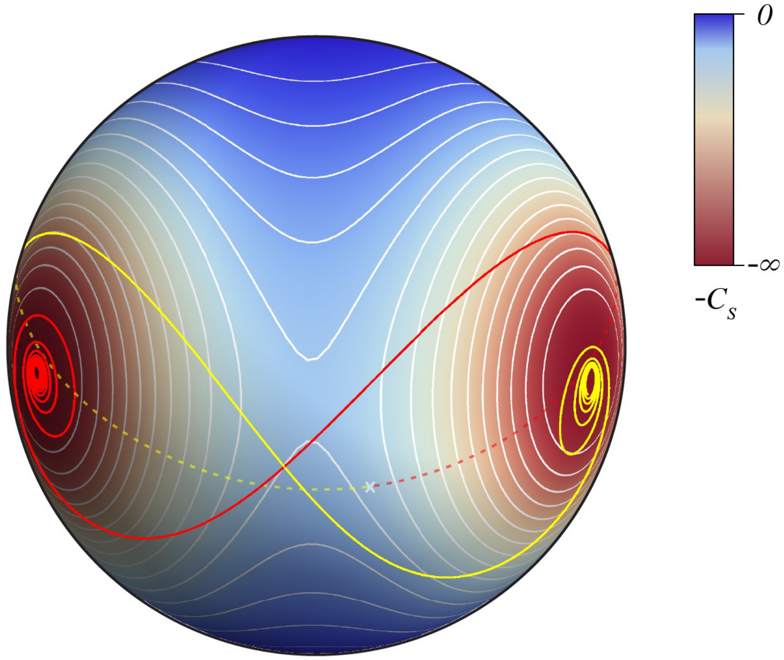




Weaker effect for Kepler-pair eccentricity. **The theory is predictive.**
For $N > 3$ many quantities can be predicted $\Rightarrow$
observations can tell us how typical our universe is.

Typicality of States: Current Entaxy

Define the **current entaxy** of a solution as
the entaxy it would have were it at its Janus point.



Due to the strong attractors on S and gravitational clustering, the complexity grows away from $D = 0 \Rightarrow$ **current entaxy decreases**. Probability for the universe to be created randomly in its current state always decreases from the maximum probability very near but not exactly at the most uniform state.

Emergent Second Law for Subsystems in All Universes

• Attractors on S + gravity enforce formation of increasingly isolated clusters. Their sizes and life times can be measured by Kepler pairs (emergent rods and clocks). The cluster gravitational self-potentials act as 'confining boxes' $\Rightarrow$ no need for Gibbs' configuration restriction.

• Each cluster has $E < 0$ and is metastably bound: once it forms and virializes, ordinary statistical quantities like Gibbs or Boltzmann entropy can be defined.

• The clusters are **branch systems** (Reichenbach 195?, Davies 1974): they avoid the time-reversal and recurrence objections to Boltzmann's H theorem because created in the past of internal observers.

• **Second Law holds:** coarse-grained descriptions assign increasing Boltzmann entropy to subsystems (Padmanabhan 1990, Gross 2002). Uniformity near J and pervasive time asymmetry either side of J ensures all arrows of time point the same way everywhere and everywhen for internal observers.

Checklist for General Relativity

The N-Body model is a **Proof of Principle**. Issues for GR:

- Attractors on S for expanding universe: Yes.
- Bounded measures: No because field theories have infinitely many dofs. Planck-type quantum blackbody resolution may 'turn up'.
- Generic 'shape + direction' Cauchy data: Yes for vacuum GR (JB & Ó Murchadha 2010), with matter perhaps.
- Existence of J: Yes at maximum expansion in Big-Bang–Big-Crunch universes, but we need it at Big Bang, which is special but there analogue of D, the York time $Y \to \infty$. Smooth continuation of conformal geometry through Big Bang would be sufficient (scale factor is gauge). **Much work in progress.**